Neutrino Astrophysics

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Neutrino Mass and Flavor Mixing in Astrophysics

Early Universe / Cosmology

Gravitational Collapse / Supernovae

Three (active) "Flavors" of Neutrinos: $\nu_e \quad \bar{\nu}_e \quad \nu_\mu \quad \bar{\nu}_\mu \quad \nu_\tau \quad \bar{\nu}_\tau$

(six species; Majorana or Dirac)

Three Families of Elementary Particles:

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix}$$

Neutrinos are electrically neutral spin-1/2 particles that interact through the Weak Interaction.

Electromagnetic Cross Section: $\gamma + e^- \rightarrow \gamma + e^-, \quad \sigma \sim 10^{-24}\ \mathrm{cm}^2$

Weak Cross Section: $\bar{\nu}_e + p \rightarrow n + e^+, \quad \sigma \sim 10^{-44}\ \mathrm{cm}^2$

$$\sigma \sim G_F^2 E^2$$

Direct Laboratory Limits on Neutrino Rest Masses

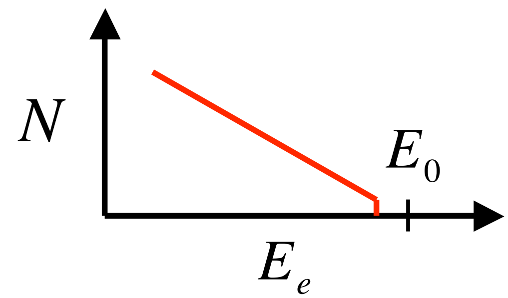
"m_{ν_τ}" < 18.2 MeV (τ - decay; Groom *et al.*, Eur. J. Phys., C15, 1, 2000.)

"m_{ν_μ}" <190 keV (π - decay)

"m_{ν_e}" < 2 eV (Tritium endpoint; J. Bonn *et al.*, Nucl. Phys. B 91, 273, 2001.)

$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$

$$\frac{dN}{dE_e} \propto \sqrt{\left(E_e - E_0\right)^2 - "m_{\nu_e}^2"}$$



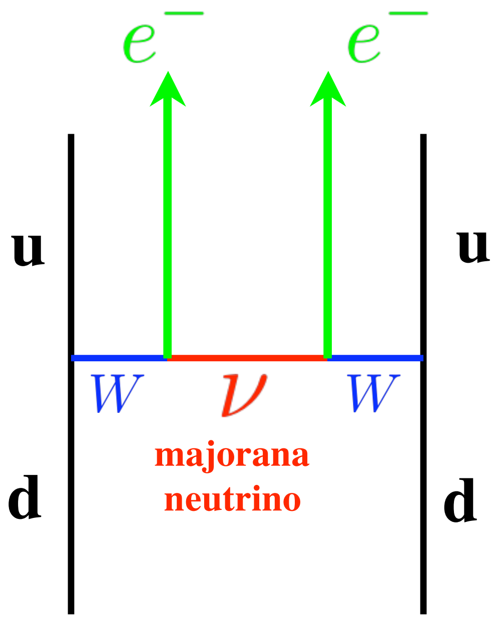
$$"m^2_{\nu_e}" \approx +0.6 \pm 2.8 \pm 2.1 \ \mathrm{eV}^2$$
$$\approx -1.6 \pm 2.5 \pm 2.1 \ \mathrm{eV}^2$$

< 4 eV² with high confidence

In terms of matrix elements of the Unitary Transformation:

$$"m^2_{\nu_e}" = m_1^2\left|U_{e1}\right|^2 + m_2^2\left|U_{e2}\right|^2 + m_3^2\left|U_{e3}\right|^2 + \ldots + m_n^2\left|U_{en}\right|^2$$

Neutrinoless Double Beta Decay

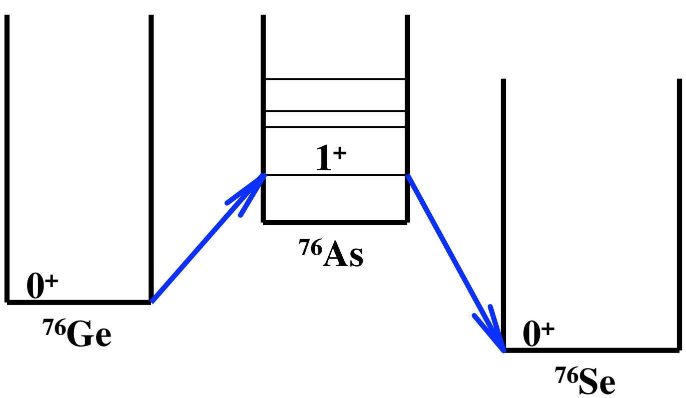


two neutrons change
into two protons

$$\Gamma_{0\nu} = \frac{1}{\tau_{\beta\beta}} = G_{0\nu} |M_{\mathrm{nuc}}|^2 m_{\beta\beta}^2$$

$$\langle m_{\beta\beta} \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

Second order weak process:
coherent sum over intermediate nuclear states



Majorana: 180 kg ^{76}Ge, after 3 years should get to > 10^{27} year lifetime, or

$$m_{\beta\beta} < 50\,\mathrm{meV}$$

Prospects for better Neutrino Data

The Lab

Tritium endpoint experments
(e.g., KATRIN $m_\nu < 200$ meV)

$0\nu2\beta$-decay
(ultimate sensitivity $m_\nu \sim 10$ meV)

New reactor expts. (θ_{13} ?)

Long Baseline expt.s (CP-violating phase)

mini-BooNE / nu-SNS

The Cosmos

Light element abundances/BBN

Supernova nucleosynthesis

Supernova neutrino signal

Large Scale Structure (LSS)
(on small scales) and/or CMB
considerations
e.g., weak lensing, CMB
(Kaplinghat, Knox, Song 04)
ultimate limit $m_\nu < 40$ meV

LSS doesn't measure m_ν, but rather the collisionless damping scale
convolved with the underlying power spectrum and growth of small scale structure.

e.g., we must ASSUME a neutrino energy spectrum to get a neutrino mass

The weak interaction, or flavor basis is not coincident with the energy eigenstate, or mass basis.

These bases are related through a unitary transformation,

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle$$

where the flavors are $\alpha = e, \mu, \tau, s, s', \ldots$

and where the mass states are $i = 1, 2, 3, 4, \ldots$

$U_{\alpha i}$ **is parameterized by vacuum mixing angles and CP-violating phases, in general.**

If we consider only two-by-two neutrino mixing then the unitary transformation is parameterized by a single vacuum mixing angle:

$$|\nu_\alpha\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

$$|\nu_\beta\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

Difference of the squares of the neutrino mass eigenvalues:

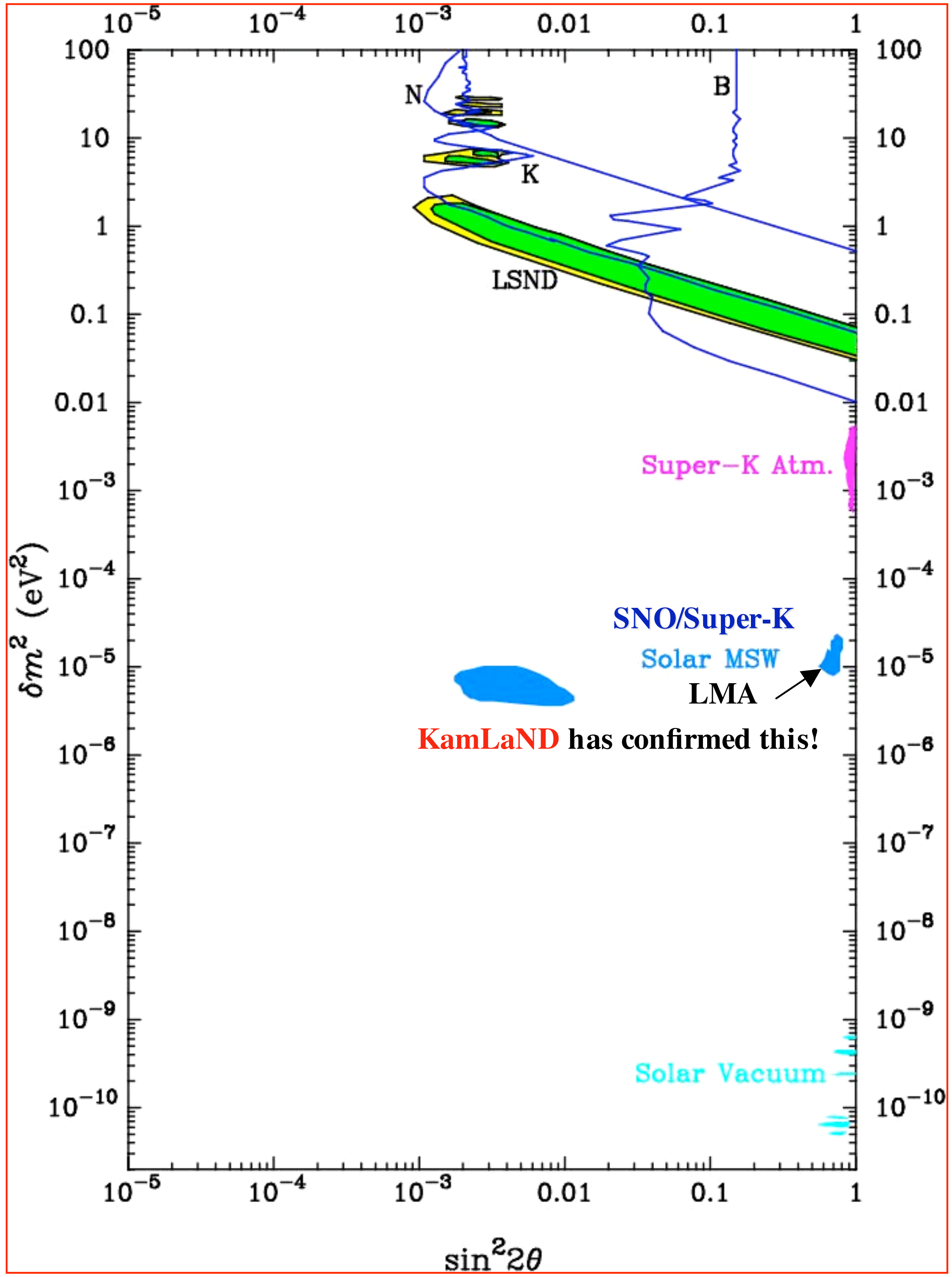
$$\delta m^2 = m_2^2 - m_1^2$$

The Experimental Neutrino Mass/Mixing Plot

LSND $\nu_\mu \Leftrightarrow \nu_e$

Atmospheric $\nu_\mu \Leftrightarrow \nu_{\tau,s?}$

Solar $\nu_e \Leftrightarrow \nu_{\mu,\tau,s?}$



Ignore LSND…

Experiment/Observation now has given us much mass/mixing data!

$\nu_\mu/\nu_\tau/\nu_e$

ν_3

$\delta m^2 \approx 3\times 10^{-3}\,\mathrm{eV}^2$

ν_2

ν_1

$\delta m^2 \approx 8\times 10^{-5}\,\mathrm{eV}^2$

(or inverted mass hierarchy)

(near) maximal mixing between ν_μ/ν_τ

also between $\nu_\mu/\nu_\tau/\nu_e$

only θ_{13} and CP-violating phase
and the absolute masses remain to be determined in this case

$$\boxed{\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U_m \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}}$$

$$\boxed{U_m = U_{23} U_{13} U_{12}}$$

$$U_{23} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

$$U_{13} \equiv \begin{pmatrix} \cos\theta_{13} & 0 & e^{i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}$$

$$U_{12} \equiv \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4 parameters

Atmospheric Neutrinos

$$\delta m^2_{23} \approx 2.5 \times 10^{-3} \text{ eV}^2$$
$$\sin^2 2\theta_{23} \approx 1.0$$

"Solar"/KamLaND Neutrinos

$$\delta m^2_{\text{sol}} \approx 8 \times 10^{-5} \text{ eV}^2$$
$$\tan^2 \theta_{12} \approx 0.42 \leftrightarrow 0.45$$

Chooz

Chooz limit on $\theta_{13} \Rightarrow$

$|U_{e3}|^2 < 2.5\%$ or $\sin^2 2\theta_{13} < 0.1$ $(\theta_{13} < \frac{\pi}{20} \approx 9^\circ)$

plus KamLaND $\Rightarrow$

$\sin^2 2\theta_{13} < 6.65 \times 10^{-2}$ $(< 0.2 \text{ at } 3\sigma)$

Nuclear Reactions in the Sun and Associated Neutrino Production

(pp neutrinos ~ 90%)

$p + p \rightarrow {}^2H + \nu_e + 0.42$ MeV

99.75 %

(pep neutrinos)

$p + e^- + p \rightarrow {}^2H + \nu_e + 1.44$ MeV

0.25 %

${}^2H(p, \gamma)\,{}^3He + 5.49$ MeV

86 %

${}^3He + {}^3He \rightarrow \alpha + 2p + 12.86$ MeV

14 %

${}^3He + \alpha \rightarrow {}^7Be + \gamma + 1.59$ MeV

99.89 %

${}^7Be + e^- \rightarrow {}^7Li + \nu_e + 14.6$ MeV

(^{7}Be neutrinos ~ 7%)

${}^7Li + p \rightarrow \alpha + \alpha + 17.35$ MeV

0.11%

${}^7Be(p, \gamma)\,{}^8B + 0.14$ MeV

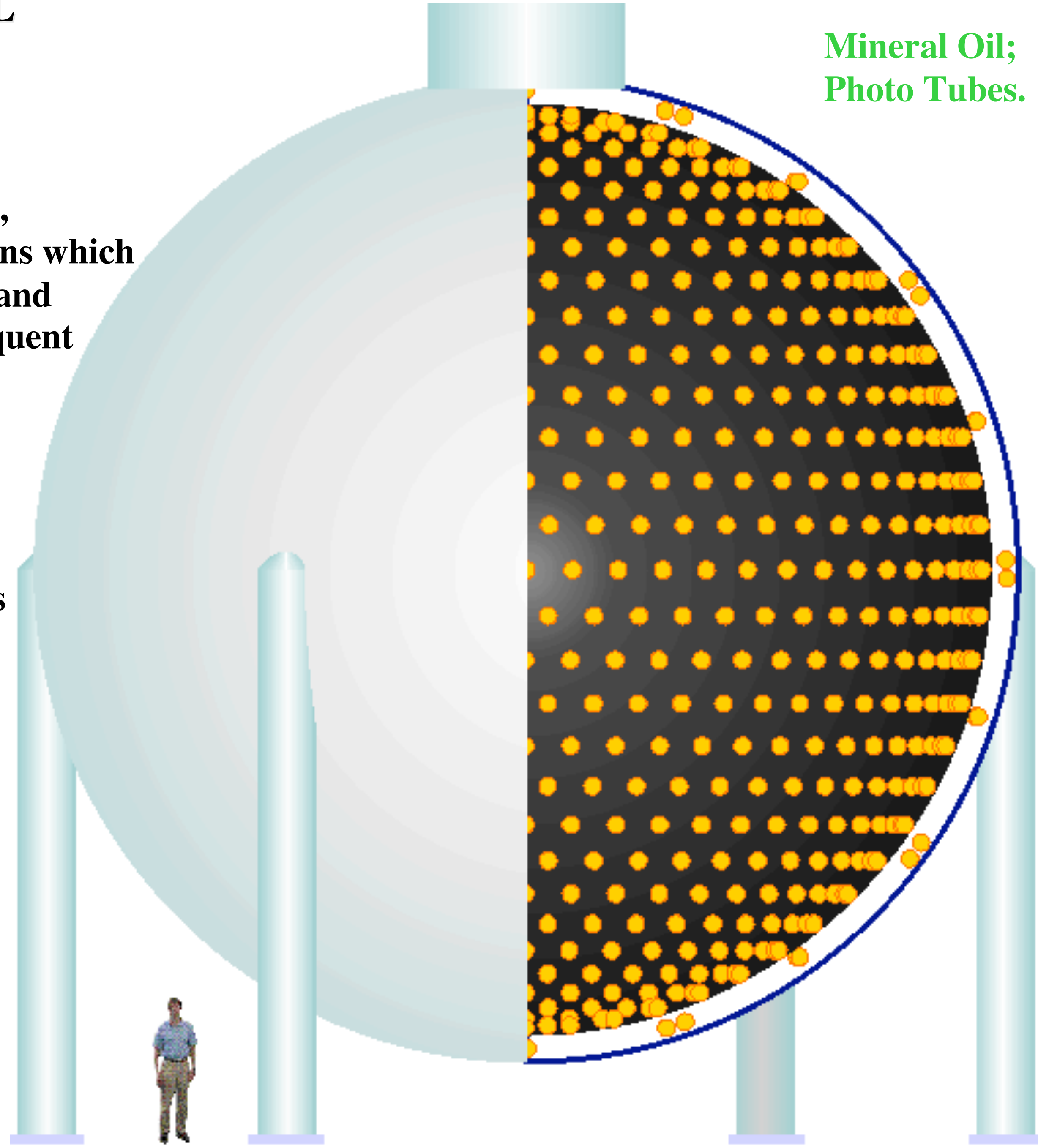
${}^8B \rightarrow {}^8Be + e^+ + \nu_e + 14.6$ MeV

(^{8}B neutrinos ~ 0.01%)

${}^8Be \rightarrow \alpha + \alpha + 3$ MeV

But, Two-Thirds of Neutrinos Coming from the Sun are Measured to have Mu and Tau Flavor!

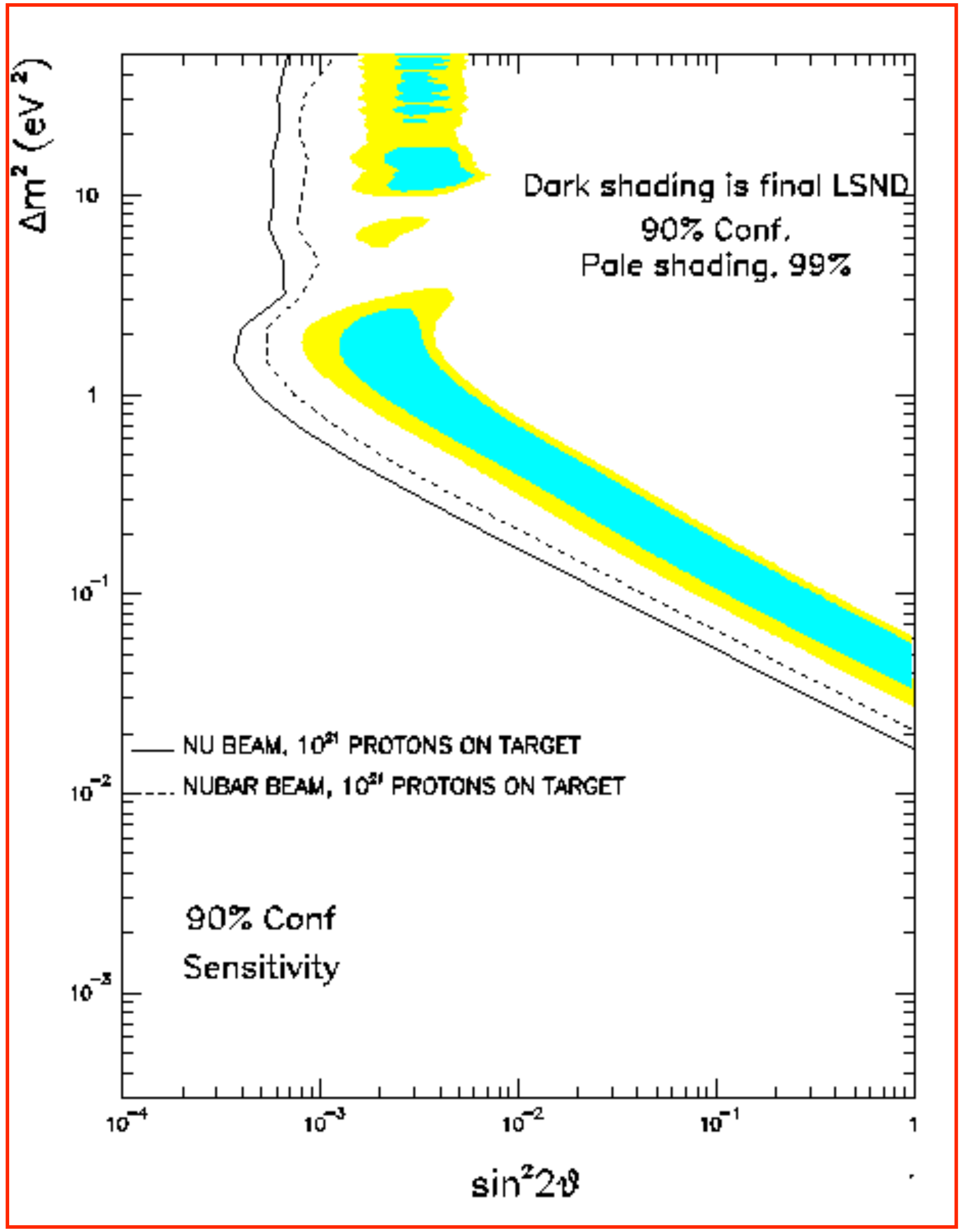
mini-BooNE at FNAL



Proton beam from FNAL booster, runs into target and produces pions which pass through a horn that focuses and selects a beam of π^- or π^+. Subsequent decay in flight produces beams of high energy muon neutrinos and anti muon neutrinos. Look for appearance of electron flavor neutrinos as an indication of vacuum oscillations. Easily covers old LSND mixing parameter space and at least some of the additional parameter space of interest in Supernovae, BBN, and Cosmology.

Expect results soon!!

Mini-BooNE explores astrophysically important mixing parameter space well beyond that probed by LSND.



All of the data cannot be explained by 3 neutrinos

The solar, atmospheric, and LSND data Imply 3 disparate values of δm^2

Either

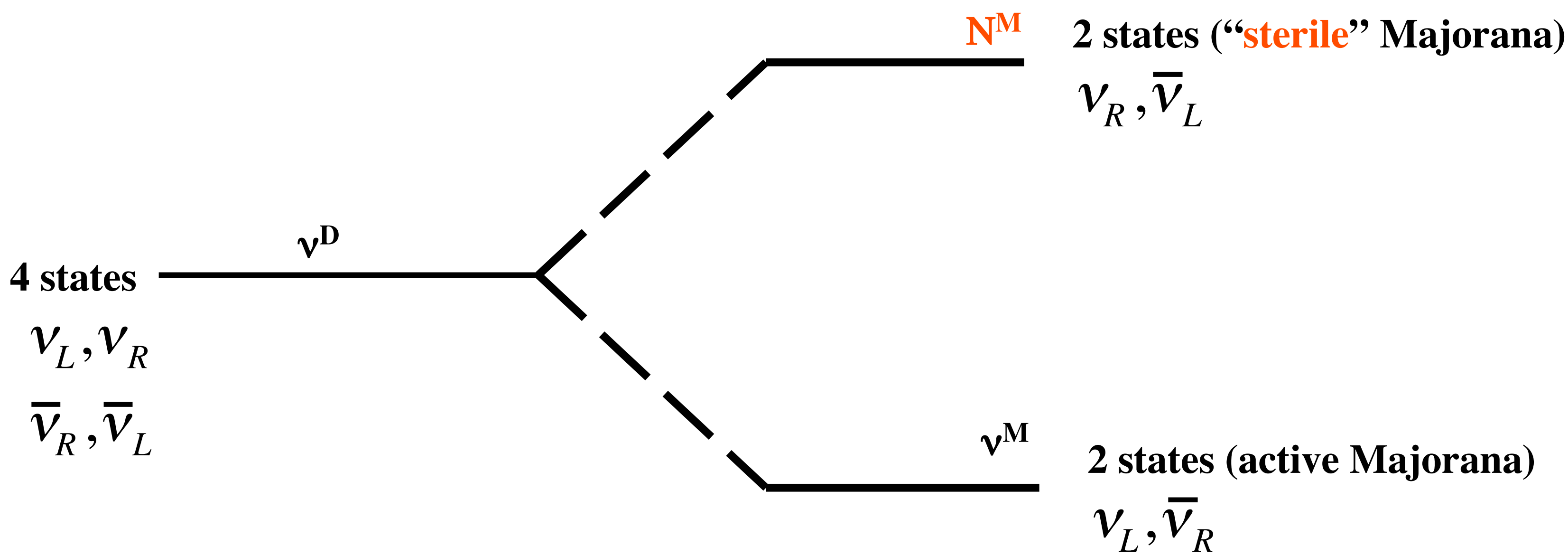
LSND signal is not a result of neutrino oscillations

And/Or

We must introduce a fourth neutrino species which, on account of the Z^0-width limit, must be "sterile," that is, an SU(2) singlet.

Why Are Neutrinos So Light?

Dirac Neutrinos	$\nu \neq \overline{\nu}$	$\nu + \overline{\nu} = 4$	**states**
Majorana Neutrinos	$\nu = \overline{\nu}$	$\nu + \overline{\nu} = 2$	**states**



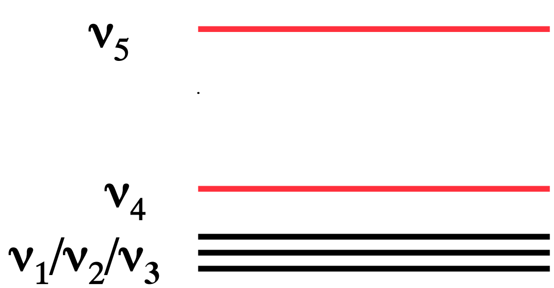
See-Saw Relation for the Product of Neutrino Masses: $(m_N)(m_\nu) \sim$ **(Really Big Mass Scale)^2**

Unification Scale?

Gell-Mann, Ramond, Slansky; Yanagida; Mohapatra & Senjanovic

(after a slide by Boris Kayser)

Sorel, Conrad, Shaevitz hep-ph/0305255 argue that a "3+2" fit is better.



$m_2' \approx 4.6\ \mathrm{eV}$ $\left\{ \begin{array}{l} U_{e5} \approx 0.07 \\ \delta m_{51}^2 \approx 21.5\ \mathrm{eV}^2 \end{array} \right.$

$\left. m_1' \sim 1\ \mathrm{eV} \right\}$

"LSND"

$U_{e4} \approx 0.12$

$\delta m_{41}^2 \approx 0.91\ \mathrm{eV}^2$

A cynic would say that more parameters always make for a better fit, but if there is one light sterile why wouldn't there be others?

Sterile Neutrinos are like cockroaches.

If you find one there are likely to be others!

What if there is a "light" sterile? Are there others?

ν_6 $m_3' \approx 32\ \text{keV}$ $\sin^2 2\theta \approx 10^{-12}$

How do we know this isn't true?

ν_5 $m_2' \approx 4\ \text{keV}$ $\sin^2 2\theta \approx 10^{-10}$

ν_4 $m_1' \approx 1\ \text{eV}$ "LSND" $\sin^2 2\theta \approx 10^{-3}$

$\nu_1/\nu_2/\nu_3$

A guess at masses for lightest neutrinos (mostly active)

$$\begin{cases} m_1 \approx 2\times10^{-6}\ \text{eV} \\ m_2 \approx 7.7\times10^{-3}\ \text{eV} \\ m_3 \approx 6.3\times10^{-2}\ \text{eV} \end{cases}$$

?

by virtue of mixing with active neutrino species,
"sterile" neutrinos are not really sterile. . .

ordinary weak interaction strength $\sim G_{\mathrm{F}}^2$

sterile neutrino interaction strength $\sim G_{\mathrm{F}}^2 \sin^2 2\theta$

For example, Dark Matter candidate sterile neutrinos have

$$\sin^2 2\theta \sim 10^{-10}$$

so it would be virtually impossible to detect these in a lab!

Cosmology

number density for fermions (+) and bosons (-)

$$dn \approx g \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{e^{E/T-\eta} \pm 1} \approx \frac{g}{2\pi^2}\left(\frac{d\Omega}{4\pi}\right)\frac{E^2 dE}{e^{E/T-\eta} \pm 1}$$

where the pencil of directions is $d\Omega = \sin\theta\, d\theta\, d\phi$

The energy density is then

$$d\varepsilon \approx \frac{g}{2\pi^2}\left(\frac{d\Omega}{4\pi}\right)\frac{E \cdot E^2 dE}{e^{E/T-\eta} \pm 1}$$

now get the total energy density by integrating over all energies and directions (relativistic kinematics limit)

$$\rho \approx \frac{T^4}{2\pi^2}\int_0^\infty \frac{x^3\, dx}{e^{x-\eta} \pm 1}$$

degeneracy parameter (chemical potential/temperature)

$$\eta \equiv \frac{\mu}{T}$$

in extreme relativistic limit

$\eta \to 0$

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15} \quad \text{and} \quad \int_0^\infty \frac{x^3\, dx}{e^x + 1} = \frac{7\pi^4}{120}$$

bosons $\rho \approx g_{\mathrm{b}} \frac{\pi^2}{30} T^4$ and fermions $\rho \approx \left(\frac{7}{8} g_{\mathrm{f}}\right)\frac{\pi^2}{30} T^4$

Statistical weight in all relativistic particles:

$$\boxed{g_{\text{eff}} = \sum_i g_i^b \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_j g_j^f \left(\frac{T_j}{T}\right)^3}$$

e.g., statistical weight in photons, electrons/positrons and six thermal, zero chemical potential (zero lepton number) neutrinos, e.g., BBN:

$$\boxed{g_{\text{eff}} = 2 + \tfrac{7}{8}\left(2 + 2 + \underset{\uparrow}{6}\right) = 10.75}$$

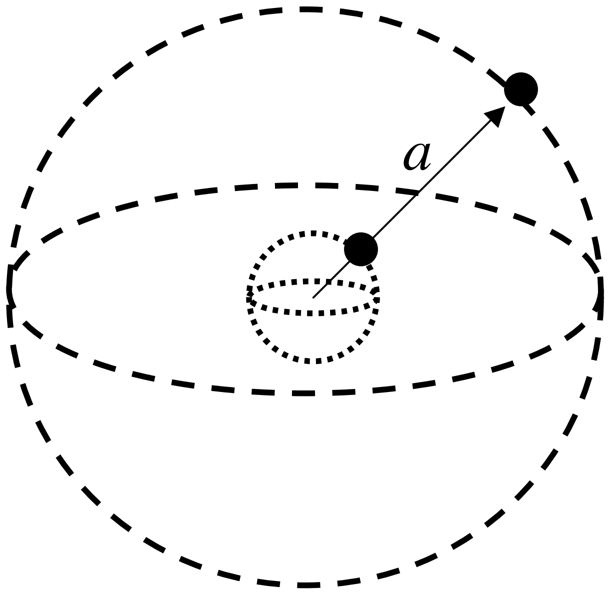
$$\boxed{\nu_e \; \bar{\nu}_e \; \nu_\mu \; \bar{\nu}_\mu \; \nu_\tau \; \bar{\nu}_\tau}$$

Homogeneity and isotropy of the universe:
implies that *total energy* inside a co-moving spherical surface is constant with time.

total energy **= (kinetic energy of expansion) + (gravitational potential energy)**

mass-energy density = ρ

test mass = *m*



$$\approx \tfrac{1}{2} m \dot{a}^2$$

$$\approx -\frac{G\left[\frac{4}{3}\pi a^3 \rho\right] m}{a}$$

$$\dot{a}^2 + k = \frac{8}{3}\pi G \rho\, a^2$$

total energy **> 0 expand forever k = -1**

total energy **= 0 for $\rho = \rho_{crit}$ k = 0**

total energy **< 0 re-collapse k = +1**

$$\Omega = \rho/\rho_{\mathrm{crit}} = \Omega_\gamma + \Omega_\nu + \underbrace{\Omega_{\mathrm{baryon}} + \Omega_{\mathrm{dark\ matter}}}_{\approx 0.3} + \Omega_{\mathrm{vacuum}} \approx 1 \quad \mathbf{(k=0)}$$

Birkhoff's Theorem

The key point in the argument on the last page was symmetry:
specifically, a homogeneous and isotropic distribution of mass and energy!

What evidence is there that this is true?

Look around you. This is manifestly NOT true on small scales. The Cosmic Microwave Background Radiation (CMB) represents our best evidence that matter is smoothly and homogeneously distributed on the largest scales.

Friedmann equation is $\dot{a}^2 + k = \frac{8}{3}\pi \mathrm{G}\rho\, a^2$ and

$\mathrm{G} = \frac{1}{m_{\mathrm{PL}}^2}$ where $\hbar = c = 1$ and the Planck Mass is $m_{\mathrm{PL}} \approx 1.22 \times 10^{22}$ MeV

radiation dominated $\rho \approx \frac{\pi^2}{30} g_{\mathrm{eff}} T^4 \sim \frac{1}{a^4}$

$\Rightarrow$ horizon is $d_{\mathrm{H}}(t) \approx 2t \approx H^{-1}$

where the Hubble parameter, or expansion rate is

$$H = \frac{\dot{a}}{a} \approx \left(\frac{8\pi^3}{90}\right)^{1/2} g_{\mathrm{eff}}^{1/2} \frac{T^2}{m_{\mathrm{PL}}}$$

$$t \approx (0.74\ \mathrm{s}) \left(\frac{10.75}{g_{\mathrm{eff}}}\right)^{1/2} \left[\frac{\mathrm{MeV}}{T}\right]^2$$

The entropy in a co-moving volume is conserved

$\Rightarrow$ $g_{\mathrm{eff}}^{1/3} a T = g_{\mathrm{eff}}'^{1/3} a' T'$ so that if the number of relativistic degrees of freedom is constant

$\Rightarrow$ $T \sim \frac{1}{a}$

Friedman-LeMaitre-Robertson-Walker (FLRW) coordinates

$$(t, r, \theta, \varphi)$$

defined through this metric . . .

$$\boxed{ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right]}$$

scale factor $a(t)$

curvature parameter $k = 1,\ 0,\ -1$

How far does a photon travel in the age of the universe?

(causal horizon)

$$\boxed{ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right]}$$

Consider a radially-directed photon ($d\theta = d\varphi = 0$)

$$d_{\mathrm{H}}(t) = \int_0^{r_{\mathrm{H}}} \sqrt{g_{rr}}\, dr = a(t) \int_0^{r_{\mathrm{H}}} \frac{dr}{\sqrt{1-kr^2}}$$

photons travel on
null world lines so $ds^2=0$ $\Rightarrow$ $$\frac{dt}{a(t)} = \frac{dr}{\sqrt{1-kr^2}}$$

$$\Rightarrow \quad d_{\mathrm{H}}(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$

Causal (Particle) Horizon

$$d_{\mathrm{H}}(t) = a(t)\int_0^t \frac{dt'}{a(t')} = \begin{cases} 2t & \text{radiation dominated} \quad a(t) \sim t^{1/2} \quad \rho \sim a^{-4} \\ 3t & \text{matter dominated} \quad a(t) \sim t^{2/3} \quad \rho \sim a^{-3} \\ H^{-1}\left[e^{Ht} - 1\right] & \begin{cases} \text{vacuum energy dominated} \\ a(t) = a(t_0)\, e^{H(t-t_0)} \\ H = \dfrac{\dot{a}}{a} = \left(\dfrac{8}{3}\pi G \rho_{\mathrm{vac}}\right)^{1/2} \end{cases} \end{cases}$$

In every case the physical (proper) distance a light signal travels goes to infinity as the value of the timelike coordinate *t* does.

Note, however, that for the vacuum-dominated case there is a finite limiting value for the FLRW radial coordinate as *t* goes to infinity . . .

$$\int_0^{r_{\mathrm{H}}} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^t \frac{dt'}{a(t')}$$

$$r_{\mathrm{H}} \approx H^{-1}\left(1 - e^{-Ht}\right) \rightarrow H^{-1} \text{ as } t \rightarrow \infty$$

some significant events/epochs in the early universe

Epoch	T	g_{eff}	Horizon Length	Mass-Energy (solar masses)	Baryon Mass (solar masses)
Electroweak phase transition	100 GeV	~100	~ 1 cm	~ 10^{-6} (~ earth mass)	~ 10^{-18}
QCD	100 MeV	51 - 62	20 km	~ 1	~ 10^{-9}
weak decoupling	2 MeV	10.75	~ 10^{10} cm	~ 10^{4}	~ 10^{-3}
weak freeze out	0.7 MeV	10.75	~ 10^{11} cm	~ 10^{5}	~ 10^{-2}
NSE	100 keV	10.75	~ 10^{13} cm (~ 1 A.U.)	~ 10^{6}	~ 1
e^-/e^+ annihilation	~ 20 keV	3.36	~ 10^{14} cm	~ 10^{8}	~ 100
photon decoupling	0. 2 eV	-	~ 350 kpc	~ 10^{18} dark matter	~ 10^{17}

1 solar mass $\approx 2\times 10^{33}$ g $\approx 10^{60}$ MeV

observed vacuum energy density

$$\rho_{\mathrm{vac}} \approx \left(3.9\ \frac{\mathrm{keV}}{\mathrm{cm}^3}\right)\left(\frac{h}{0.71}\right)^2\left(\frac{\Omega_{\mathrm{vac}}}{0.73}\right)$$

$$\approx (2.3\ \mathrm{meV})^4\left(\frac{h}{0.71}\right)^2\left(\frac{\Omega_{\mathrm{vac}}}{0.73}\right)$$

neutrino mass scale ?

Leptogenesis

Generate net lepton number through CP violation in the neutrino sector.

Transfer some of this or a pre-existing net lepton number to a net baryon number.

Contribution of one thermal (zero chemical potential) neutrino flavor to the closure fraction:

$$\Omega_\nu \mathrm{h}^2 \approx \left(\frac{m_\nu}{92\mathrm{eV}}\right)\left(\frac{T_\gamma}{2.75\ \mathrm{K}}\right)^3$$

where the Hubble expansion rate parameter is h = H_0/100 km s^{-1} Mpc^{-1}

for example, at minimum the mass of the heaviest active neutrino is the square root of δm^2_{atmos} ~ 3 10^{-3} eV^2 and at most all the active neutrinos have a mass ~ 1 eV so generously the contribution to closure of the active neutrinos plausibly could be between a few percent and one hundred fifty percent of the baryon rest mass closure fraction:

$$6.5 \times 10^{-2} \geq \Omega_\nu \geq 1.2 \times 10^{-3}$$

Contribution to closure of all neutrino species with thermal (black body, BB) energy spectra.
A thermal energy spectrum is characterized by a temperature and a degeneracy parameter (chemical potential divided by temperature).

$$\Omega_{\nu_{\mathrm{tot}}} \mathrm{h}^2 \approx \left(5.31\times 10^{-3}\right)\left(\frac{T_\gamma}{2.725\ \mathrm{K}}\right)^3 \left[\sum_{i_{\mathrm{BB}}} \left(\frac{\mathrm{F}_2\left(\eta_{\nu_i}\right)}{\frac{3}{2}\zeta(3)}\right)\left(\frac{T_{\nu_i}}{\left(\frac{4}{11}\right)^{1/3} T_\gamma}\right)^3 \left(\frac{"m_{\nu_i}"}{1\ \mathrm{eV}}\right)\right]$$

e.g., a neutrino and antineutrino with mass

$$m_{\nu_3} \approx \sqrt{\delta m^2_{\mathrm{atm}}} \approx 0.055\ \mathrm{eV}$$

$$\Rightarrow \quad \Omega_{\nu_{\mathrm{tot}}} \approx (0.0012)\left(\frac{0.7}{\mathrm{h}}\right)^2 \quad \Rightarrow \quad \sim 3\%\ \text{of baryon rest mass}$$

Relativistic Fermi Integral of order *k*

$$F_k\left(\eta\right) \equiv \int_0^\infty \frac{x^k}{e^{x-\eta}+1}\,dx$$

For example, $F_2\left(0\right) = \frac{3}{2}\zeta\left(3\right)$ $\qquad$ $F_3\left(0\right) = \frac{7\pi^4}{120}$

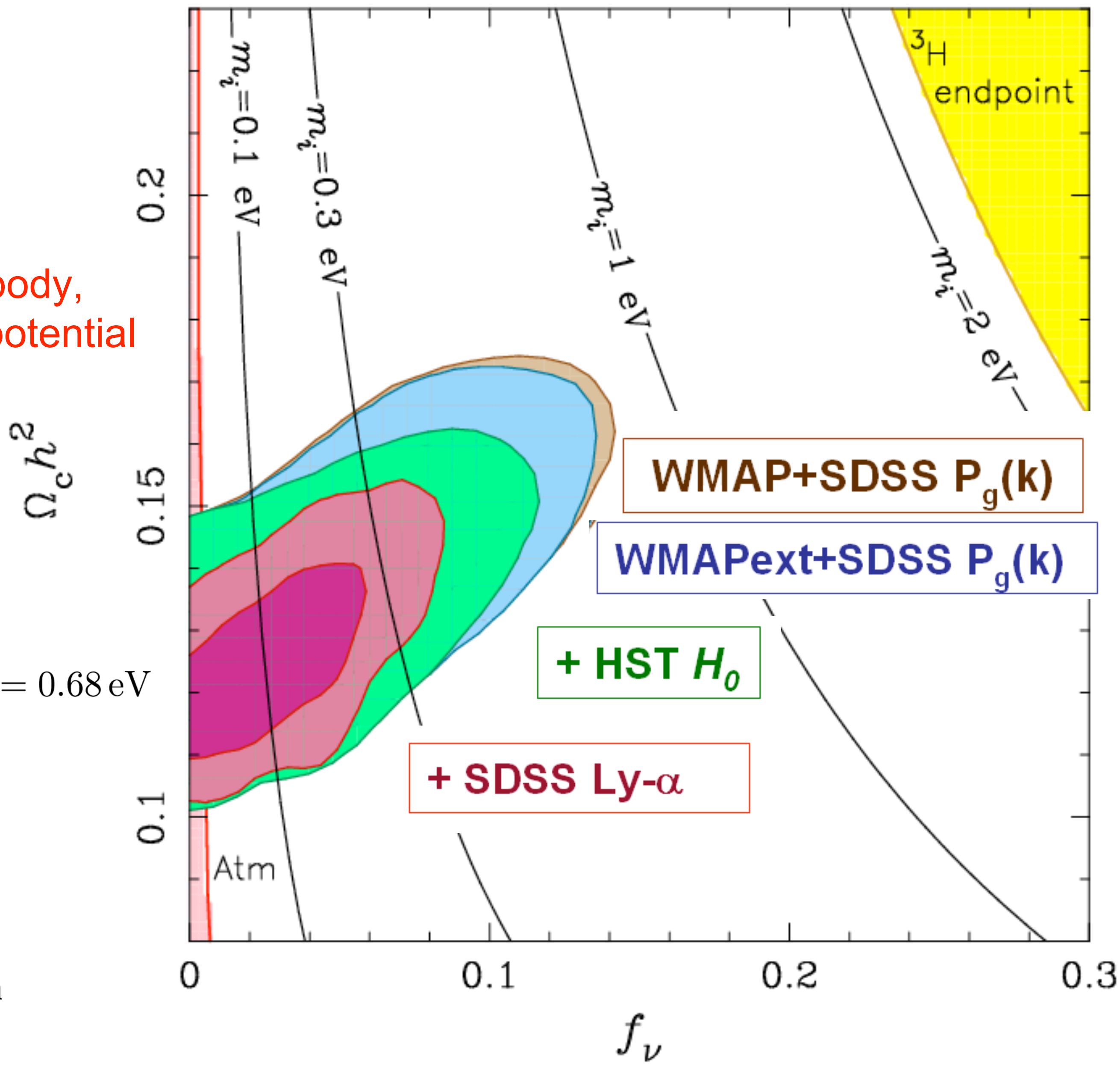
and $\qquad \zeta\left(3\right) \approx 1.20206$

cosmological constraints on neutrino rest mass - rule out LSND?

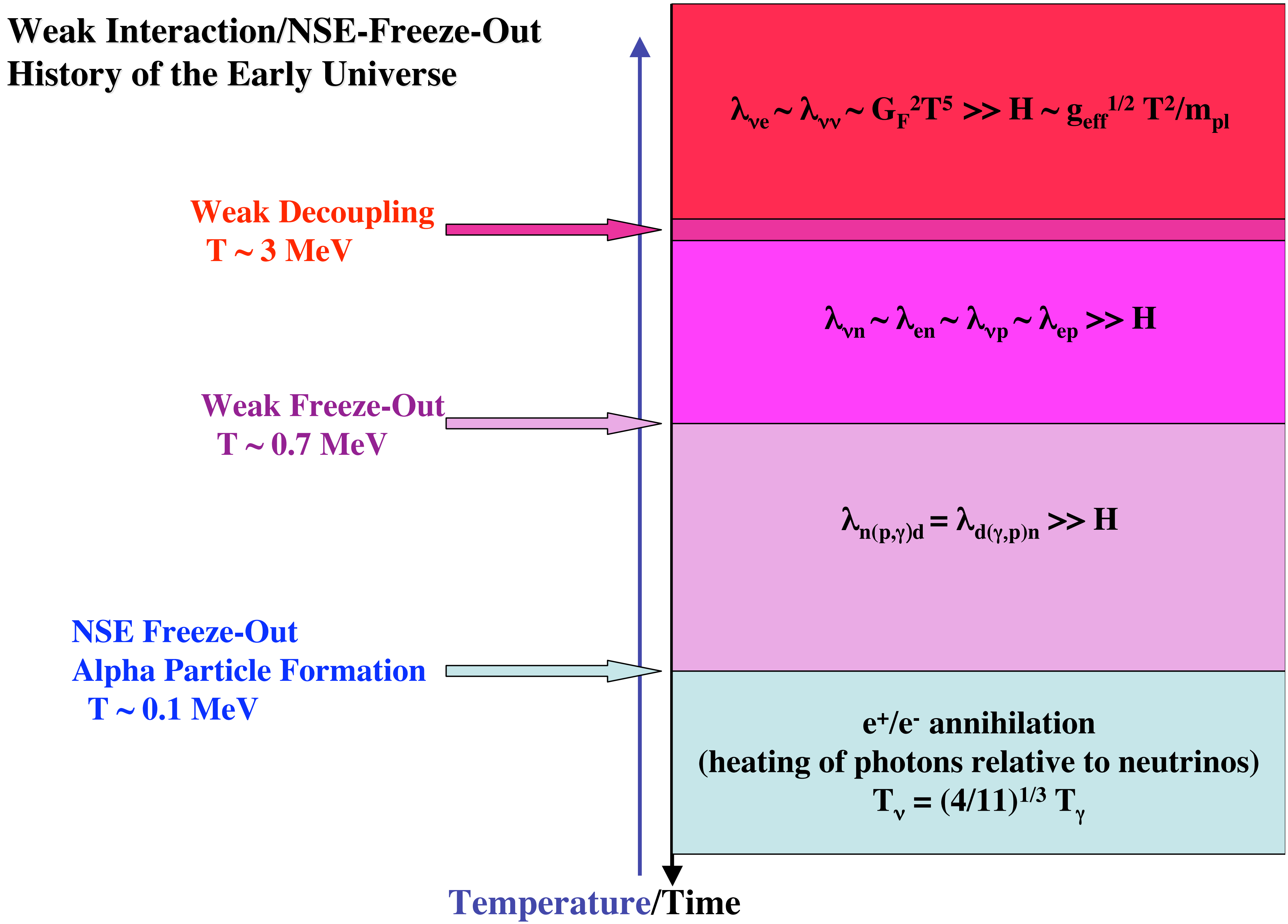
WMAP$_{\text{+ACBAR+CBI}}$ + SDSS + HST: ν Dark Matter

assumes that neutrinos have thermal, black body, zero chemical potential energy spectra

WMAP $\sum m_\nu = 0.68\,\text{eV}$



K. Abazajian



Weak Interaction/NSE-Freeze-Out
History of the Early Universe
$\lambda_{\nu e} \sim \lambda_{\nu\nu} \sim G_F{}^2T^5 >> H \sim g_{eff}{}^{1/2}\, T^2/m_{pl}$
Weak Decoupling
T ~ 3 MeV
$\lambda_{\nu n} \sim \lambda_{en} \sim \lambda_{\nu p} \sim \lambda_{ep} >> H$
Weak Freeze-Out
T ~ 0.7 MeV
$\lambda_{n(p,\gamma)d} = \lambda_{d(\gamma,p)n} >> H$
NSE Freeze-Out
Alpha Particle Formation
T ~ 0.1 MeV
e^+/e^- annihilation
(heating of photons relative to neutrinos)
$T_\nu = (4/11)^{1/3}\, T_\gamma$
Temperature/Time

Weak Decoupling

This occurs when the rates of neutrino scattering reactions on electrons/positrons drop below the expansion rate.

After this epoch the neutrino gas ceases to efficiently exchange energy with the photon-electron plasma.

neutrino scattering rate $\lambda_\nu \sim \left(\mathrm{G}_\mathrm{F}^2\, T^2\right)\left(T^3\right) = \mathrm{G}_\mathrm{F}^2\, T^5$

where the Fermi constant is $\mathrm{G}_\mathrm{F} \approx 1.166 \times 10^{-11}\ \mathrm{MeV}^{-2}$

expansion rate $H \approx \left(\frac{8\pi^3}{90}\right)^{1/2} g_\mathrm{eff}^{1/2}\, \frac{T^2}{m_\mathrm{PL}}$

weak decoupling temperature

$$T_\mathrm{WD} \approx \left(\frac{8\pi^3}{90}\right)^{1/6} \frac{g_\mathrm{eff}^{1/6}}{\left(\mathrm{G}_\mathrm{F}^2\, m_\mathrm{PL}\right)^{1/3}} \approx 1.5\ \mathrm{MeV} \left(\frac{g_\mathrm{eff}}{10.75}\right)^{1/6}$$

Weak Freeze-Out (WFO)

. . . occurs when the rates of these weak reactions drop below the expansion rate of the universe . . .

$$\bar{\nu}_e + p \leftrightarrow n + e^+$$

$$\nu_e + n \leftrightarrow p + e^-$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

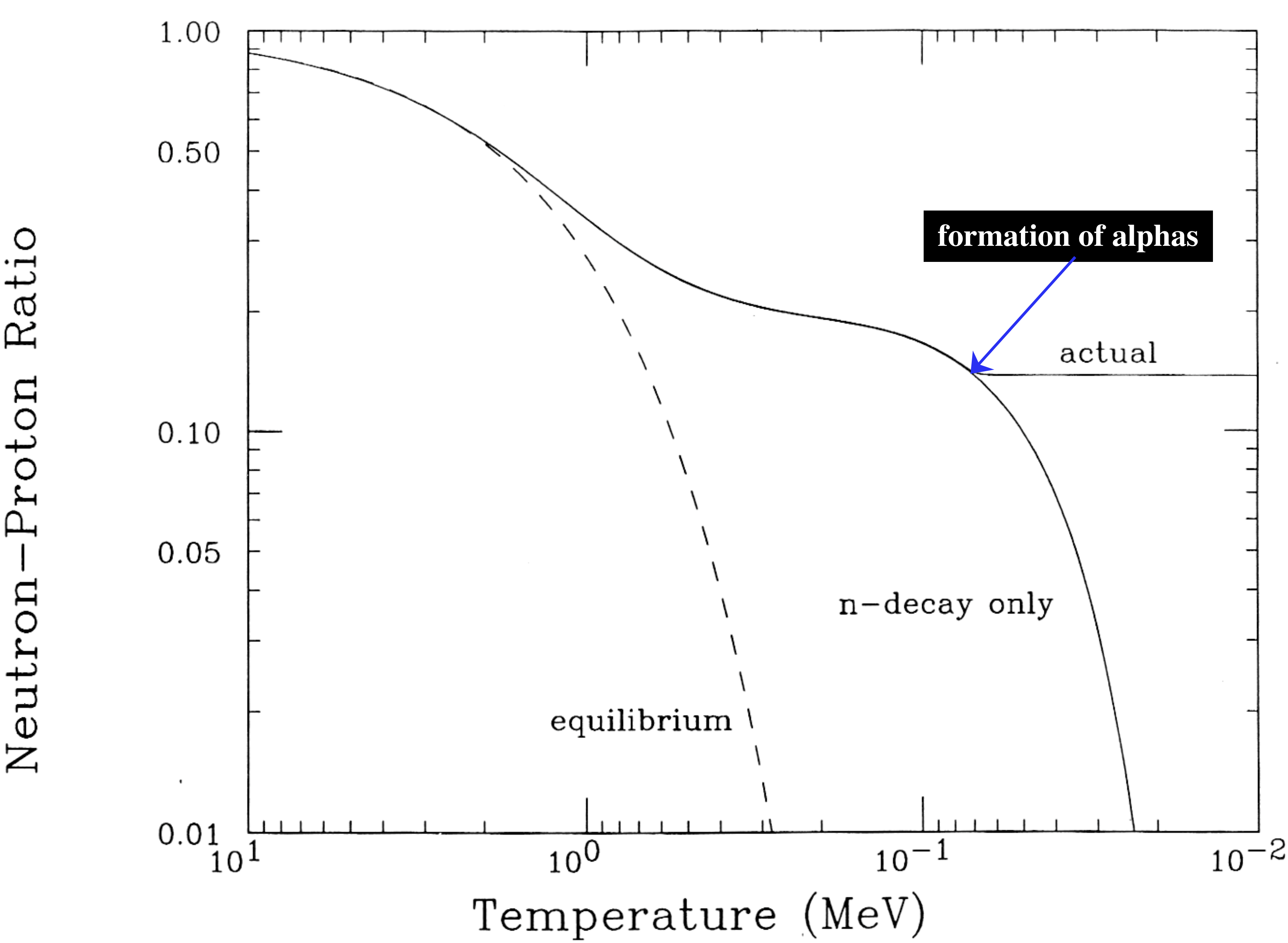
Only true when leptons possess thermal distribution functions.

$$n/p = (\lambda_{\bar{\nu}ep} + \lambda_{pe-}) / (\lambda_{\nu en} + \lambda_{ne+} + \lambda_{n\text{-decay}}) = e^{-(\delta m_{np}/T)-\eta_e}$$

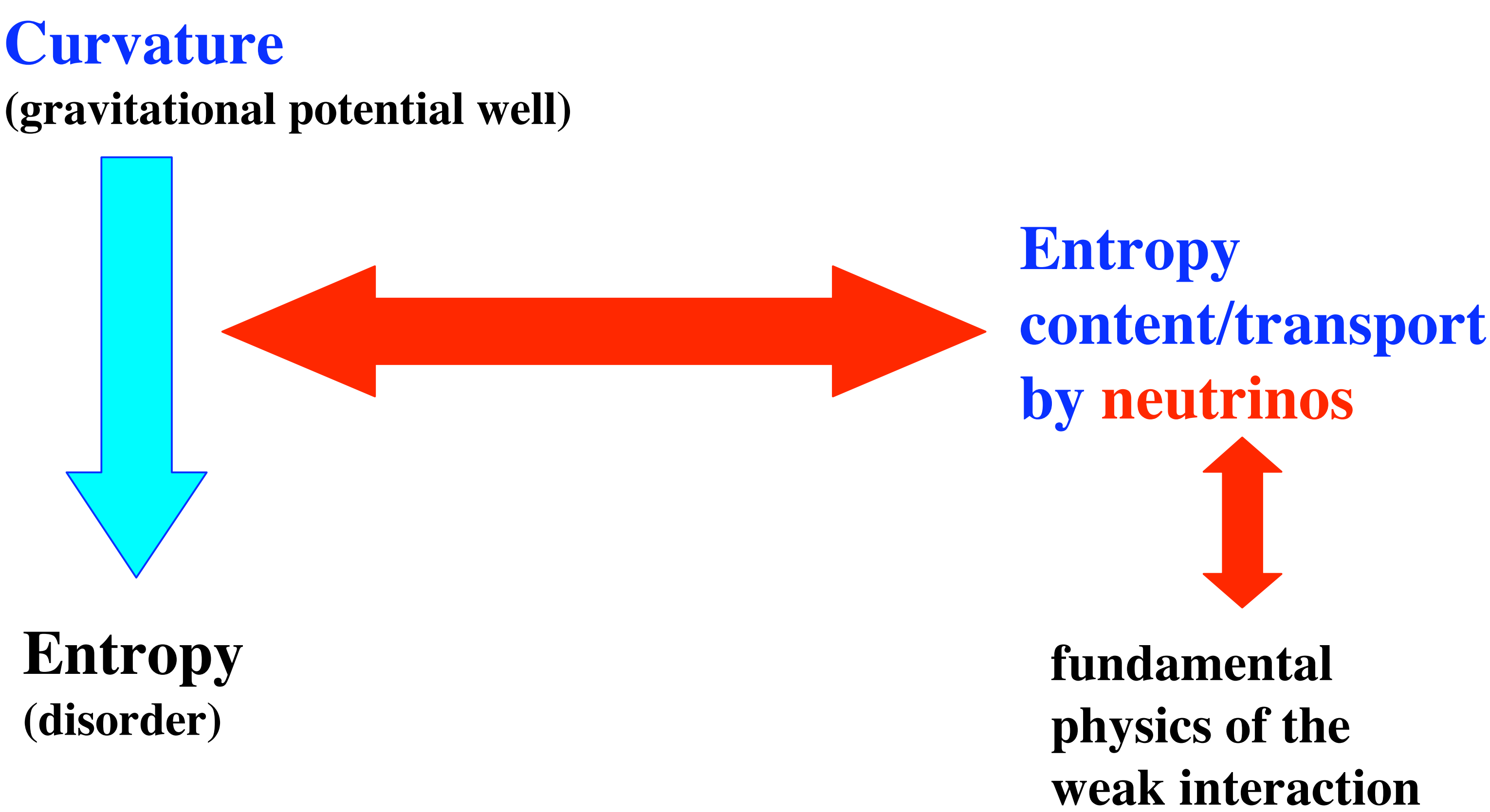
where the neutron-proton mass difference is $\delta m_{np} \sim 1.29$ MeV

and where the electron neutrino degeneracy parameter is $\eta_e = \mu_e/T$.

$T_{WFO} = 0.7$ MeV, and if we take $\eta_e = 0$, we get $n/p\big|_{WFO} \sim 1/6$


formation of alphas
actual
n−decay only
equilibrium
1.00
0.50
0.10
0.05
0.01
10^1
10^0
10^{-1}
10^{-2}
Neutron−Proton Ratio
Temperature (MeV)

There is a deep connection between spacetime curvature and entropy (and neutrinos)



Entropy

entropy per baryon (in units of Boltzmann's constant k) of the air in this room $s/\mathrm{k} \sim 10$

entropy per baryon (in units of Boltzmann's constant k) characteristic of the sun $s/\mathrm{k} \sim 10$

entropy per baryon (in units of Boltzmann's constant k) for a 10^6 solar mass star $s/\mathrm{k} \sim 1000$

entropy per baryon (in units of Boltzmann's constant k) of the universe $s/\mathrm{k} \sim 10^{10}$

total entropy of a black hole of mass M

$$S/\mathrm{k} = 4\pi\left(\frac{M}{m_{\mathrm{pl}}}\right)^2 \approx 10^{77}\left(\frac{M}{\mathrm{M}_{\mathrm{sun}}}\right)^2$$

where the gravitational constant is $G = \frac{1}{m_{\mathrm{pl}}^2}$

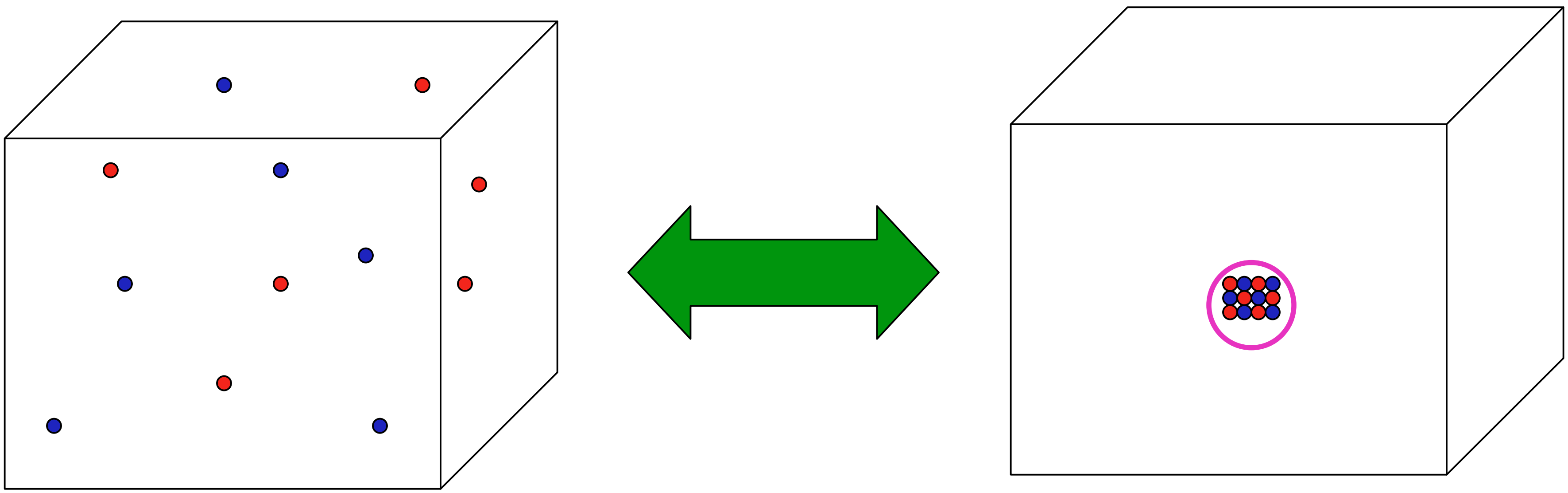
and the Planck mass is $m_{\mathrm{pl}} \approx 1.221 \times 10^{22}$ MeV

Entropy

$$S = k \log \Gamma$$

a measure of a system's disorder/order

High Entropy **Low Entropy**



12 free nucleons **^{12}C nucleus**

entropy per baryon in radiation-dominated conditions

entropy per unit proper volume

$$S \approx \frac{2\pi^2}{45} g_s\, T^3$$

proper number density of baryons $n_b = \eta\, n_\gamma$

entropy per baryon $s \approx \dfrac{S}{n_b}$

The "baryon number" is defined to be the ratio of the net number of baryons to the number of photons:

$$\boxed{\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma}}$$

The "baryon number," or baryon-to-photon ratio, η is a kind of "inverse entropy per baryon," but is not a co-moving invariant.

$$\boxed{\eta \approx \frac{2\pi^4}{45}\frac{1}{\zeta(3)}\frac{g_{total}}{g_\gamma}S^{-1}}$$

The entropy of the universe is huge

We know the entropy-per-baryon of the universe because we measure the cosmic microwave background temperature and we measure the baryon density through the deuterium abundance.

$$\boxed{S/k = 2.53 \times 10^8 \, (\Omega_b h^2)^{-1} \sim 10^{10}}$$

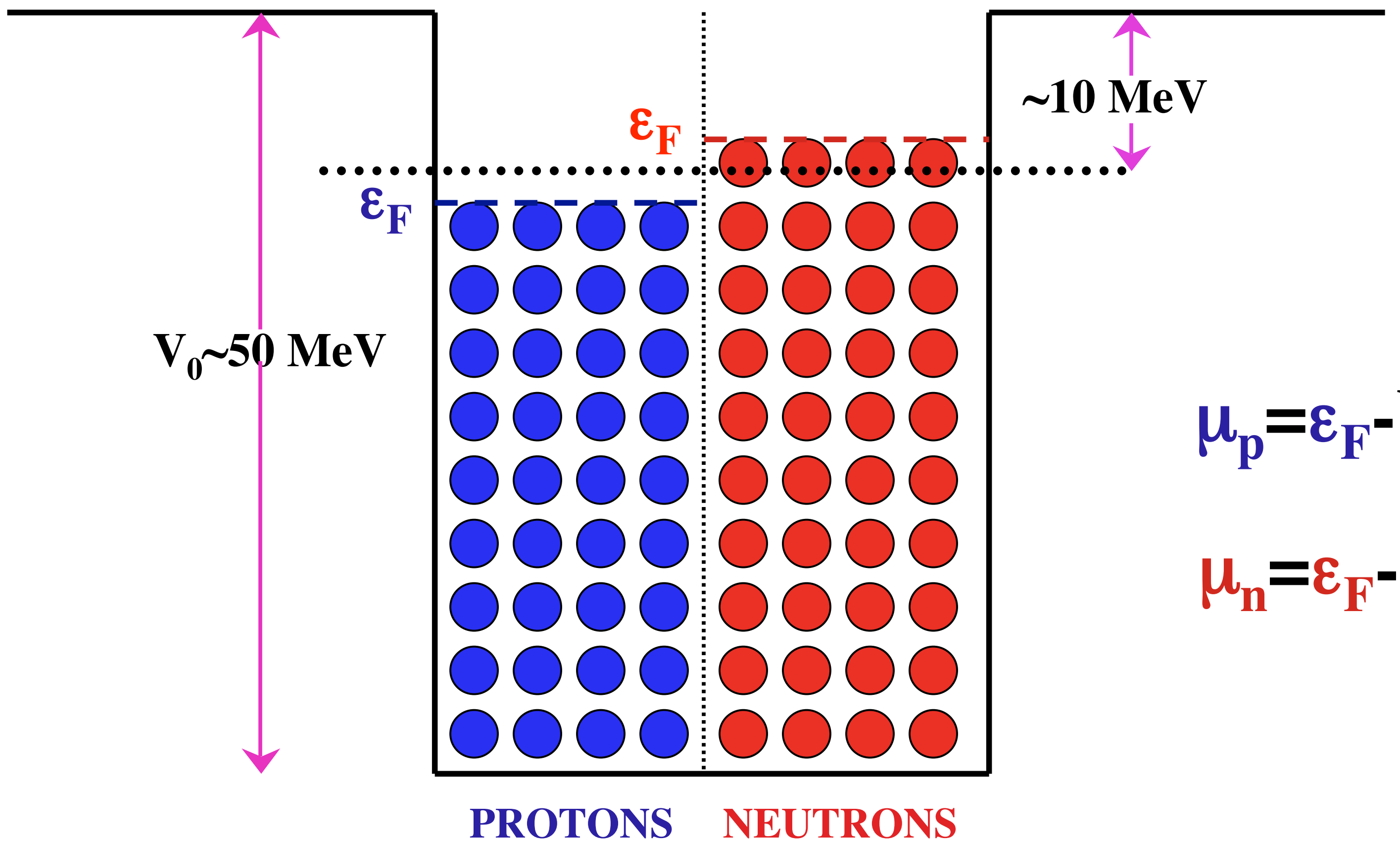
Deuterium, CMB, and large scale structure measurements imply $\Omega_b h^2 \sim 0.02$

Neglecting relatively small contributions from black holes, SN, shocks, nuclear burning, etc., S/k has been constant throughout the history of the universe.

S/k is a (roughly) co-moving invariant.

Schematic "Nucleus"

(ignore Coulomb potential for protons)



$$\mu_p = \varepsilon_F - V_0$$

$$\mu_n = \varepsilon_F - V_0$$

Thermonuclear Reaction Rates

Rate per reactant is the thermally-averaged product of flux and cross section.

$$a + X \rightarrow Y + b \quad \text{or } X(a,b)Y$$

$$\text{rate per X nucleus is } \lambda = \left(1 + \delta_{aX}\right)^{-1} \langle \sigma \, v \rangle$$

$$\sim \frac{1}{E} \exp\left(-b \frac{Z_a Z_X e^2}{\sqrt{E}} \right)$$

Rates can be very temperature sensitive, especially when Coulomb barriers are big.

At high enough temperature the forward and reverse rates for nuclear reactions can be large and equal and these can be larger than the local expansion rate. This is equilibrium. If this equilibrium encompasses all nuclei, we call it Nuclear Statistical Equilibrium (NSE).

In most astrophysical environments NSE sets in for $T_9 \sim 2$.

$$\boxed{\begin{array}{l} T_9 \equiv \dfrac{T}{10^9\ \mathrm{K}} \\ \text{where Boltzmann's constant is } k_B \approx 0.08617 \text{ MeV per } T_9 \end{array}}$$

Freeze-Out from Nuclear Statistical Equilibrium (NSE)

In NSE the reactions which build up and tear down nuclei have equal rates, and these rates are large compared to the local expansion rate.

$$Z\,p + N\,n \longleftrightarrow A(Z,N) + \gamma$$

$$\underbrace{\text{nuclear mass A is the sum of protons and neutrons } A=Z+N}$$

$$Z\,\mu_p + N\,\mu_n = \mu_A + Q_A$$

Binding Energy of Nucleus A

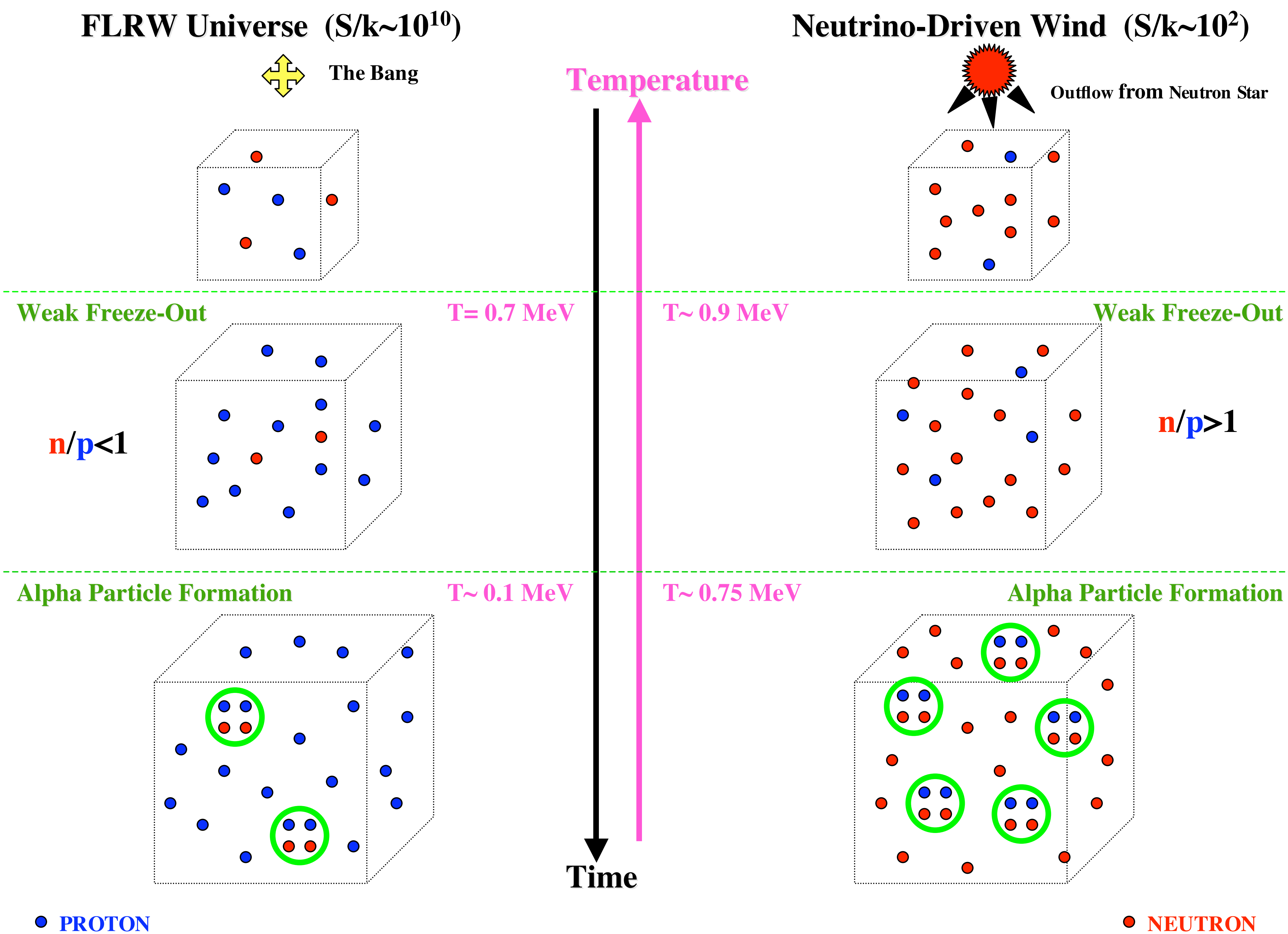
Saha Equation

$$Y_{A(Z,N)} \approx \left[S^{1-A}\right] G \pi^{\frac{7}{2}(A-1)} 2^{\frac{1}{2}(A-3)} A^{3/2} \left(\frac{T}{m_b}\right)^{\frac{3}{2}(A-1)} Y_p^Z Y_n^N e^{Q_A/T}$$

Typically, each nucleon is bound in a nucleus by ~ 8 MeV.

For alpha particles the binding per nucleon is more like 7 MeV.

But alpha particles have mass number A=4, and they have almost the same binding energy per nucleon as heavier nuclei so they are favored whenever there is a competition between binding energy and disorder (high entropy).


FLRW Universe (S/k~10^10)
The Bang
Temperature
Neutrino-Driven Wind (S/k~10^2)
Outflow from Neutron Star
Weak Freeze-Out
T= 0.7 MeV
T~ 0.9 MeV
Weak Freeze-Out
n/p<1
n/p>1
Alpha Particle Formation
T~ 0.1 MeV
T~ 0.75 MeV
Alpha Particle Formation
Time
PROTON
NEUTRON

There are two neutrons for every alpha particle, so in the limit where *every* neutron gets incorporated into an alpha particle the abundance of alpha's will be

$$Y_\alpha \approx \frac{1}{2}Y_n = \frac{1}{2}X_n \quad \text{where } Y_\alpha = X_\alpha/4$$

number density is $n_A = n_b Y_A$
and abundance is $Y_A = X_A/A$
where mass fraction is X_A
and A is nuclear mass number
and baryon number density is n_b

The alpha mass fraction at the α formation epoch, T ~ 100 keV, is then

$$X_\alpha = 4Y_\alpha \approx 2Y_n \approx \frac{2n_n}{(n_n+n_p)} = \frac{2(n_n/n_p)}{(1+n_n/n_p)}$$

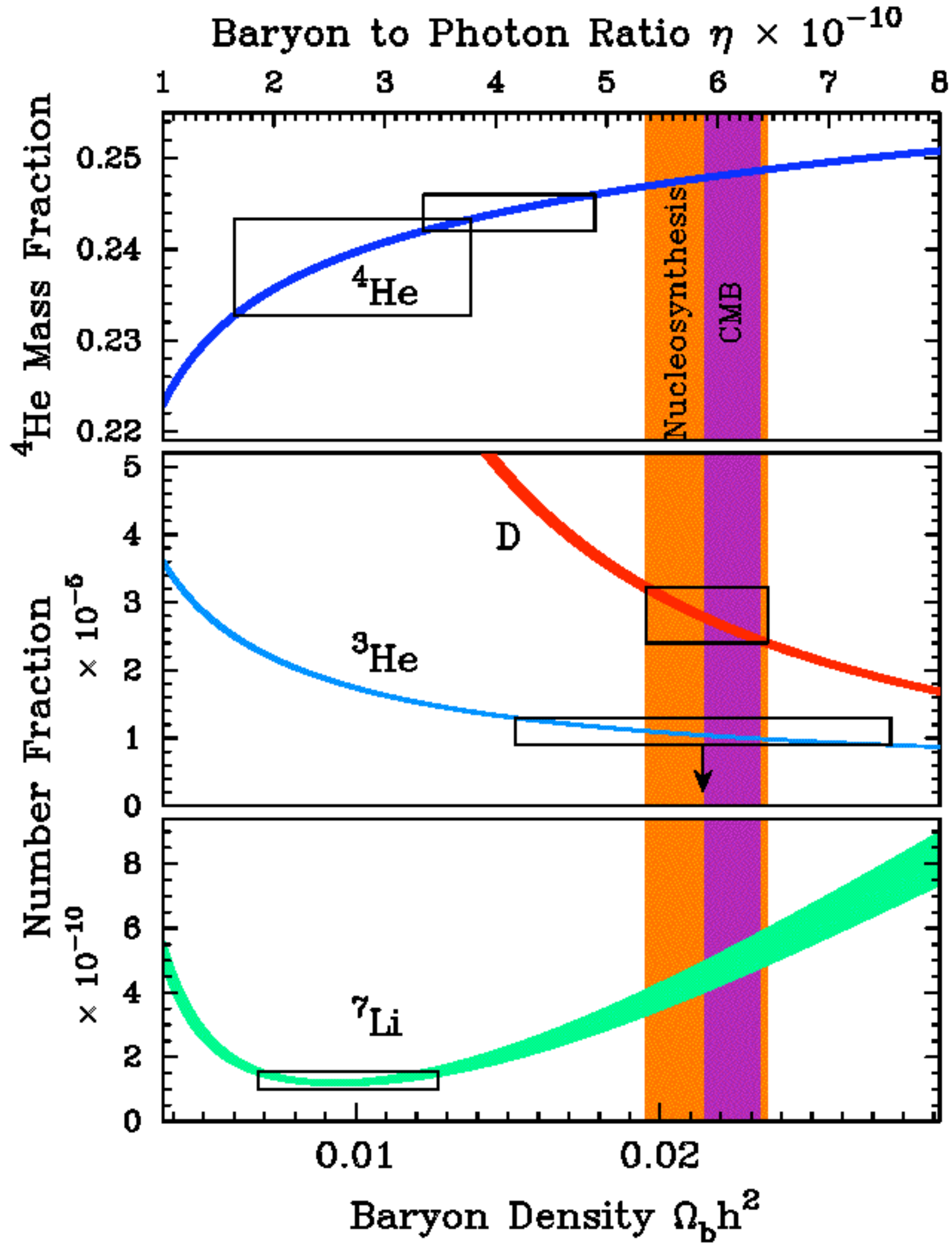
$$\approx \frac{2(1/7)}{(1+1/7)} = \frac{2/7}{8/7} = \frac{2}{8} = 0.25$$

where we have used $\frac{n_n}{n_p} \approx \frac{1}{7}$ at the time the alpha particles form

Remember that at Weak Freeze Out, $T \approx 0.7$ MeV,
the neutron to proton ratio for zero lepton number is

$$\frac{n_n}{n_p} \approx \frac{1}{6}$$

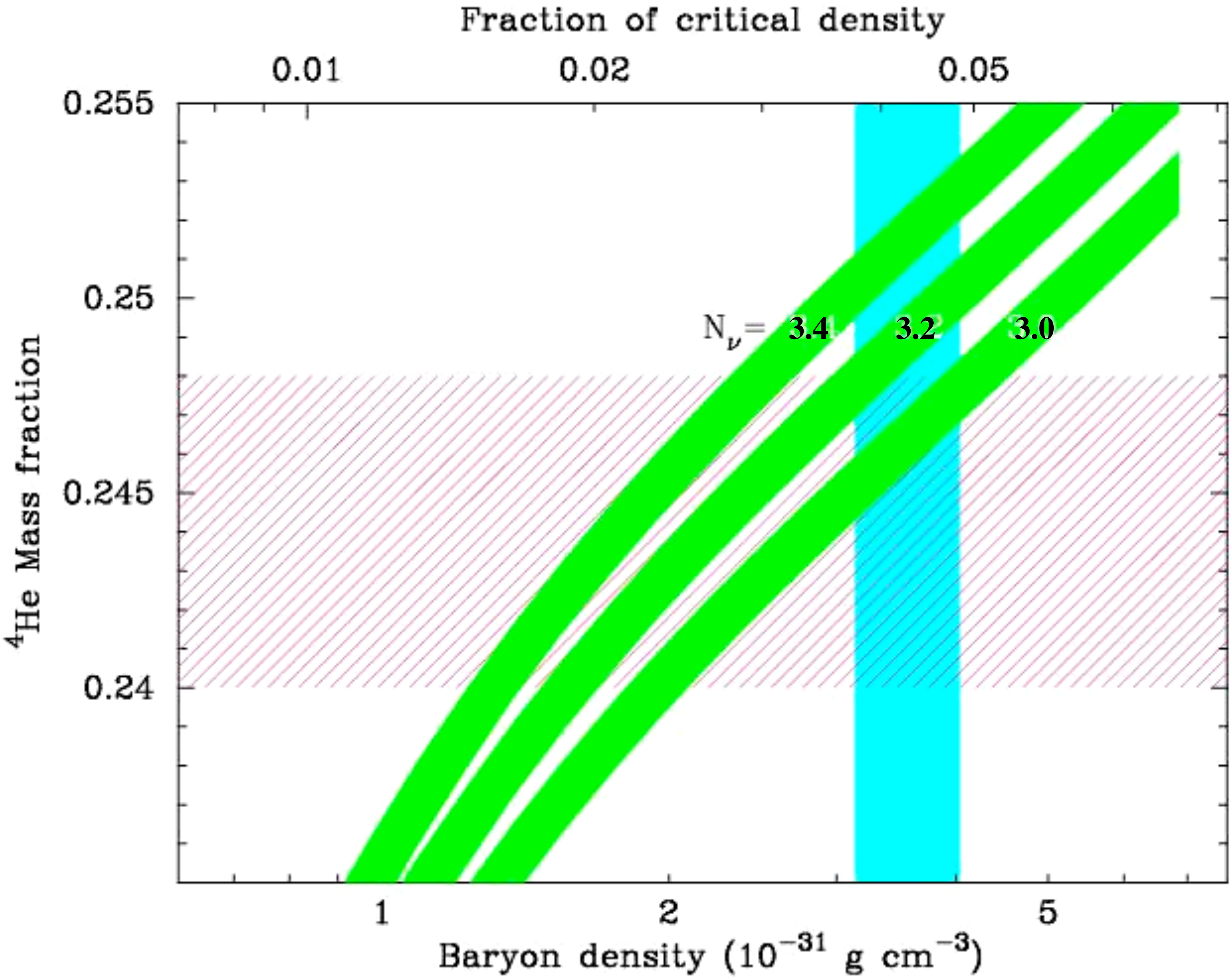
Observations of the isotope-shifted line of deuterium along the lines of sight to high redshift QSO's (Tytler group) provide an accurate determination of the baryon-to-photon ratio η. (CMB acoustic peak ratios give results consistent with these, as do considerations of large scale structure.)



This completely alters the way we look at BBN.

The "baryon number" is defined to be the ratio of the net number of baryons to the number of photons:

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma}$$



Fraction of critical density
0.01
0.02
0.05
0.255
0.25
0.245
0.24
^{4}He Mass fraction
N_ν = 3.4 3.2 3.0
1
2
5
Baryon density (10^{-31} g cm^{-3})

baryon number of universe $\longrightarrow$ $\eta \equiv \frac{n_{\rm b} - n_{\bar{\rm b}}}{n_\gamma}$

From CMB acoustic peaks, and/or observationally-inferred primordial D/H:

$$\eta \approx 6 \times 10^{-10}$$

three lepton numbers $\longrightarrow$

$$\left\{ \begin{aligned} L_{\nu_e} &\approx \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_\gamma} \\ L_{\nu_\mu} &= \frac{n_{\nu_\mu} - n_{\bar{\nu}_\mu}}{n_\gamma} \\ L_{\nu_\tau} &= \frac{n_{\nu_\tau} - n_{\bar{\nu}_\tau}}{n_\gamma} \end{aligned} \right.$$

From observationally-inferred ^{4}He and large scale structure and using *collective (synchronized) active-active neutrino oscillations* (Abazajian, Beacom, Bell 03; Dolgov et al. 03):

$$|L_{\nu_{\mu,\tau}}| \sim L_{\nu_e} < 0.15$$

Core Collapse Supernovae

Gravitational Collapse of Stars to Neutron Stars or Black Holes Releases Huge Amounts of Energy, Most of it as Neutrinos of all flavors.

$$\text{Stellar Mass Range}: \;\sim 10\,\mathrm{M}_\odot \;\text{ to }\; \sim 100\,\mathrm{M}_\odot$$

$$\rightarrow \;\text{collapse of } 1.4\,\mathrm{M}_\odot \text{ iron core}$$

$$\rightarrow \;10\% \text{ of core rest mass radiated as neutrinos}$$

$$\underbrace{\begin{array}{l}\text{Very Massive Objects} \;\sim 10^2\,\mathrm{M}_\odot \text{ to } 10^4\,\mathrm{M}_\odot \\ \text{Supermassive Objects}: \;\sim 10^4\,\mathrm{M}_\odot \text{ to } 10^8\,\mathrm{M}_\odot\end{array}}_{\boxed{\text{collapse to black hole, } \sim 5\% \text{ of rest mass radiated in neutrinos}}}$$

Nuclear Burning Stages of a 25 M_{sun} Star

Burning Stage	Temperature	Density	Time Scale
Hydrogen	5 keV	5 g cm^{-3}	7 X 10^6 years
Helium	20 keV	700 g cm^{-3}	5 X 10^5 years
Carbon	80 keV	2 X 10^5 g cm^{-3}	600 years
Neon	150 keV	4 X 10^6 g cm^{-3}	1 year
Oxygen	200 keV	10^7 g cm^{-3}	6 months
Silicon	350 keV	3 X 10^7 g cm^{-3}	1 day
Core Collapse	700 keV ↓	4 X 10^9 g cm^{-3} ↓	~ seconds of order the free fall time
"Bounce"	~ 2 MeV	~10^{15} g cm^{-3}	~milli-seconds
Neutron Star	< 70 MeV initial ~ keV "cold"	~10^{15} g cm^{-3}	initial cooling ~ 15-20 seconds ~ thousands of years

Massive Stars are Giant Refrigerators

From core carbon/oxygen burning onward
the neutrino luminosity exceeds the photon luminosity.

Neutrinos carry energy/entropy away from the core!

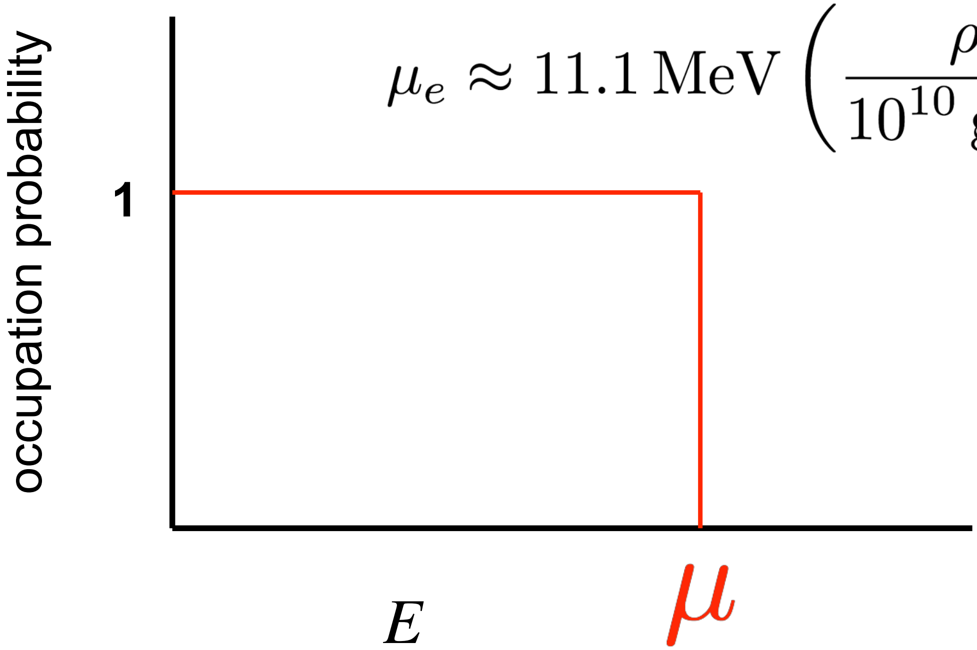
Core goes from $S/k \sim 10$ on the Main Sequence (hydrogen burning)
to a thermodynamically cold $S/k \sim 1$ at the onset of collapse!

e.g., the collapsing core of a supernova can be a
frozen (Coulomb) crystalline solid with a
temperature ~1 MeV!

degenerate electrons: large chemical potential, small T

$$\text{temperature in core collapse } T \approx 1\,\mathrm{MeV}$$

$$\mu_e \approx 11.1\,\mathrm{MeV} \left(\frac{\rho\, Y_e}{10^{10}\,\mathrm{g\,cm^{-3}}} \right)^{1/3}$$



The electron fraction is:

$$Y_e \equiv \frac{n_{e^-} - n_{e^+}}{n_{\mathrm{baryons}}}$$

The Core Collapse Supernova Phenomenon is Exquisitely Sensitive to Flavor Changing Processes and New Neutrino Physics:

- Gravitational collapse results in high electron and ν_e Fermi Energies (representing ~ 10^{57} units of e-lepton number); μ/τ charged leptons are absent and the corresponding neutrinos are pair-produced so they carry no net lepton number. Any process that changes flavor $\nu_e \rightarrow \nu_{\mu/\tau/s}$ will open phase space for electron capture as well as reducing e-lepton number.

- Later, energy (10% of the core's rest mass) is in seas of active neutrinos of all flavors. Entropy and lepton number transported by neutrinos.

- Neutron/proton ratio (crucial for nucleosynthesis) determined by electron degeneracy or by charged current neutrino capture:

$$\nu_e + n \rightleftharpoons p + e^- \qquad \bar{\nu}_e + p \rightleftharpoons n + e^+$$

Neutrinos Dominate the Energetics of Core Collapse Supernovae

Explosion
only ~1% of neutrino energy

Total optical + kinetic energy, 10^{51} ergs

Total energy released in Neutrinos, 10^{53} ergs

10% of star's rest mass!

$$E_{\mathrm{GRAV}} \approx \frac{3}{5}\frac{\mathrm{G}\,M_{\mathrm{NS}}^2}{R_{\mathrm{NS}}} \approx 3\times10^{53}\mathrm{ergs}\left[\frac{M_{\mathrm{NS}}}{1.4\mathrm{M}_{\mathrm{sun}}}\right]^2\left[\frac{10\ \mathrm{km}}{R_{\mathrm{NS}}}\right]$$

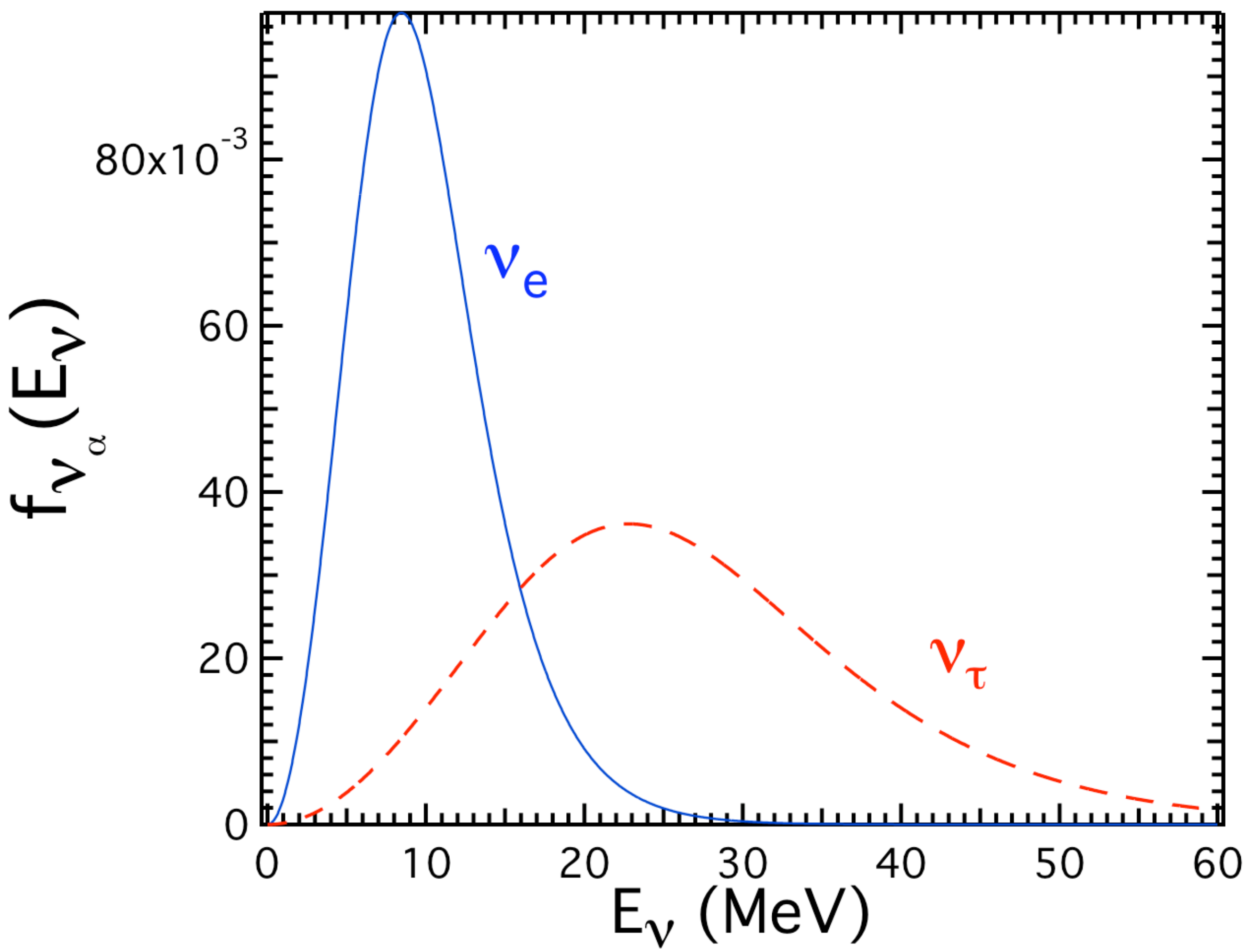
Neutrino diffusion time, $\tau_\nu \approx 2$ s **to** 10 s

$$L_\nu \approx \frac{1}{6}\frac{GM_{\mathrm{NS}}^2}{R_{\mathrm{NS}}}\frac{1}{\tau_\nu} \approx 4\times10^{51}\mathrm{ergs\ s}^{-1}$$

Neutrino Distribution Functions f_ν

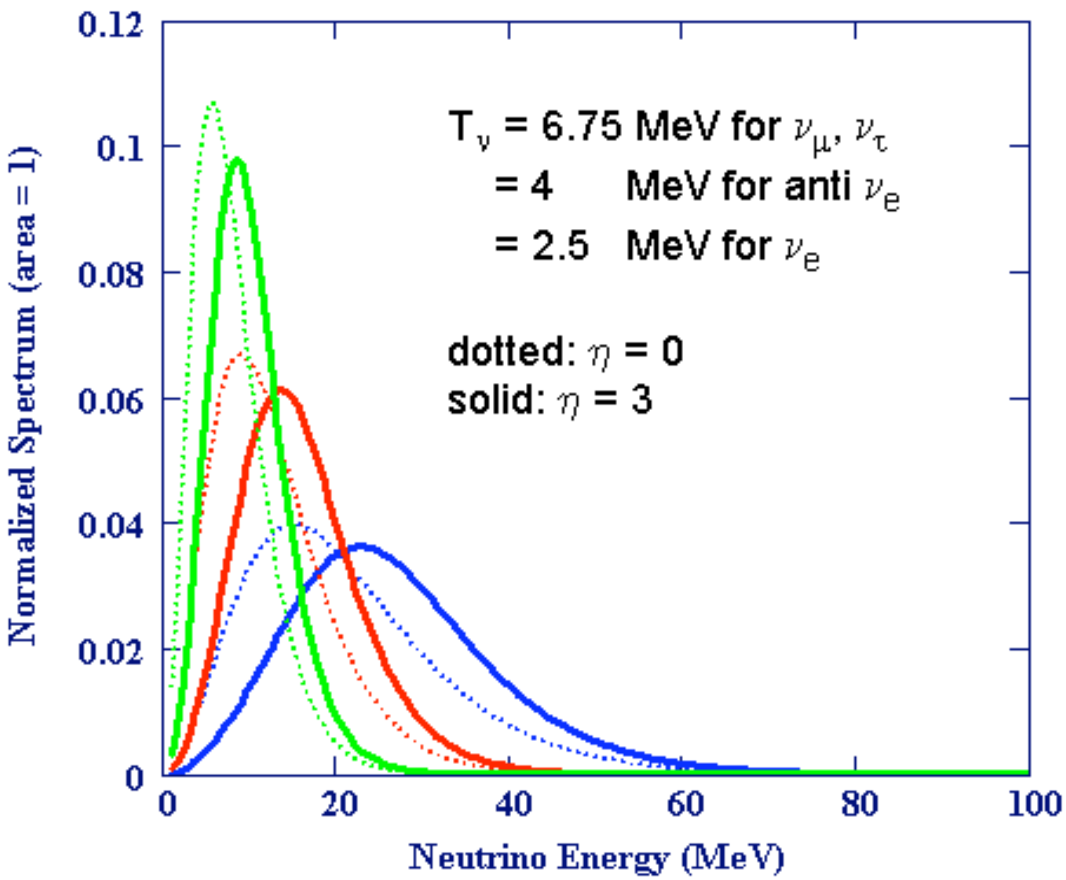
At late times (t_{pb} > 10 s) we expect an average energy hierarchy:

$$\langle E_{\nu_{\mu,\tau}} \rangle > \langle E_{\bar{\nu}_e} \rangle > \langle E_{\nu_e} \rangle$$



Neutrino Energy Spectra

- at "Neutrino Sphere"
- Near Fermi-Dirac energy distribution



$$\langle E_{\nu_e} \rangle \sim 10 \text{ MeV}$$

$$\langle E_{\bar{\nu}_e} \rangle \sim 16 \text{ MeV}$$

$$\langle E_{\nu_{\mu,\tau}} \rangle \sim 27 \text{ MeV}$$

We really do not know what the spectra/fluxes are but they are likely to be different for different flavors at *some* times of interest.

The flux of neutrinos in a pencil of directions and energies is

$$d\varphi_\nu \approx \frac{L_\nu}{\pi R_\nu^2}\left(\frac{d\Omega_\nu}{4\pi}\right)\frac{1}{\langle E_\nu \rangle} f(E_\nu)\, dE_\nu$$

The (black body) neutrino distribution function is

$$f(E_\nu) = \frac{1}{T_\nu^3 F_3(\eta_\nu)} \frac{E_\nu^2}{e^{E_\nu/T_\nu - \eta_\nu} + 1}$$

$$F_k(\eta_\nu) = \int_0^\infty \frac{x^k\, dx}{e^{x-\eta_\nu} + 1}$$

Neutron-to-proton ratio and energy deposition largely determined by these processes:

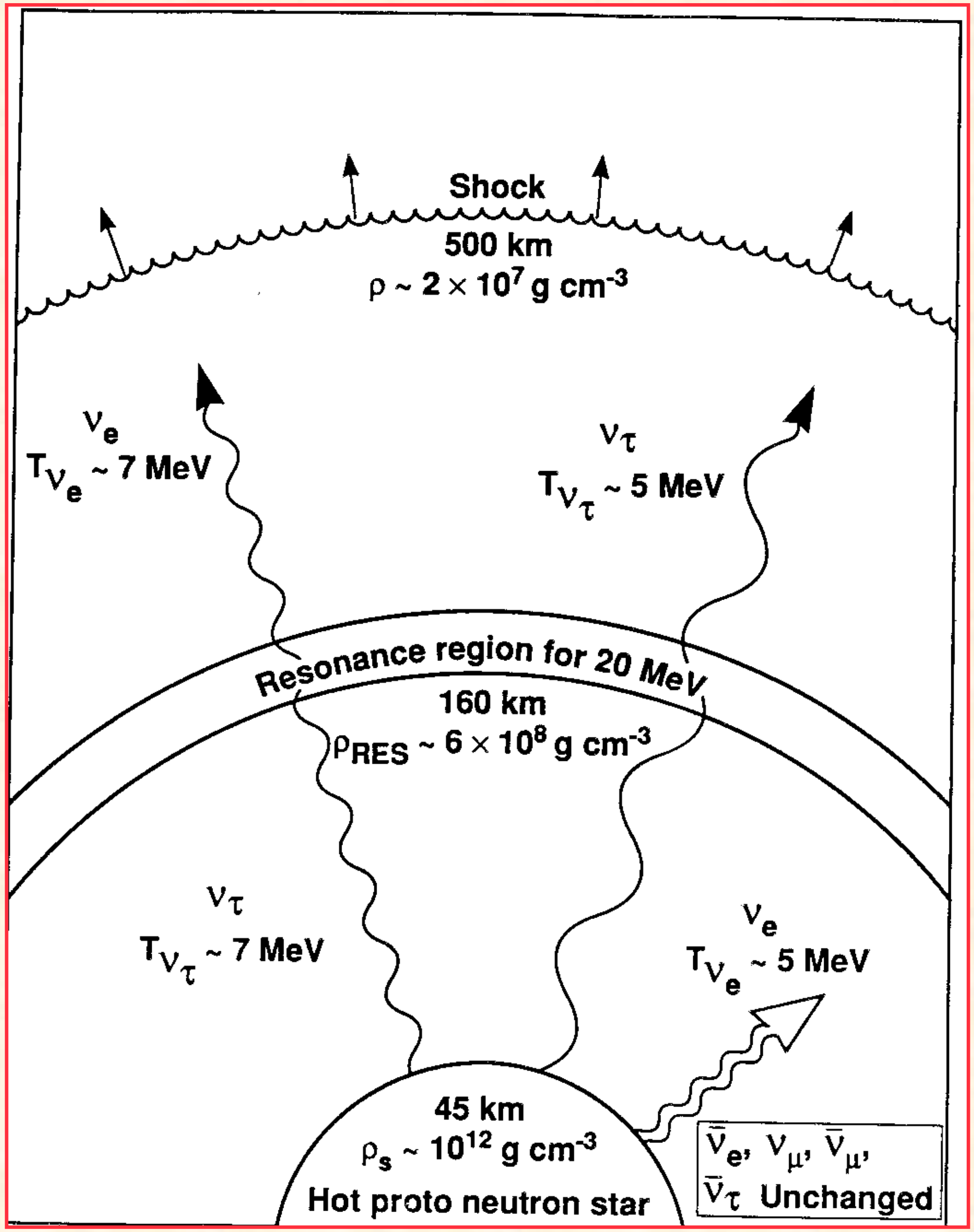
$$\boxed{\nu_e + n \longrightarrow p + e^-}$$

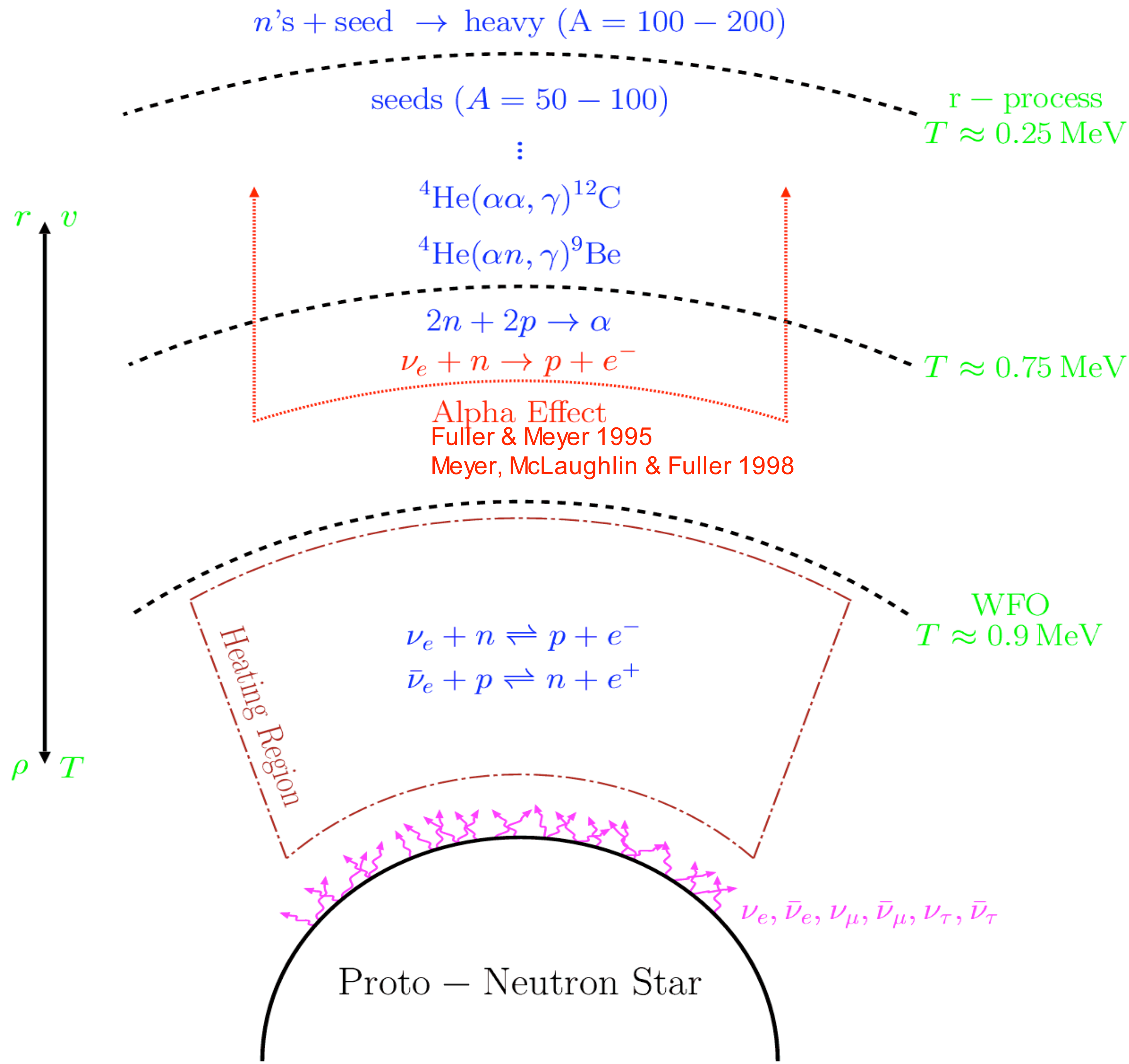
$$\boxed{\bar{\nu}_e + p \longrightarrow n + e^+}$$

Shock likely re-energized principally by neutrino processes occurring underneath shock:

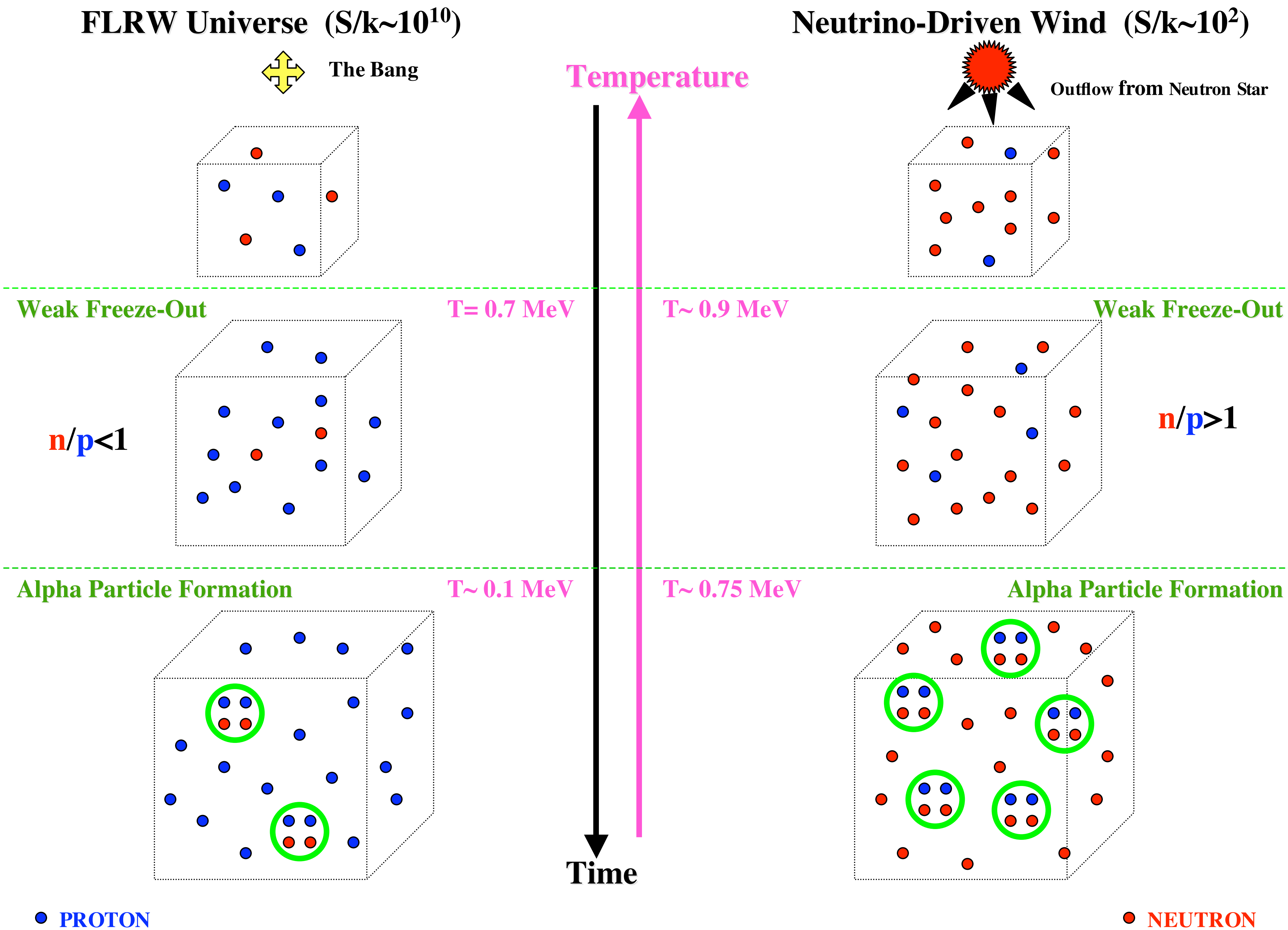
$$\nu_e + n \rightarrow p + e^-$$
$$\bar{\nu}_e + p \rightarrow n + e^+$$

Neutrino-nucleus processes, both charged and neutral current, may also be important in "pre-heating" of material ahead of shock. Can this alleviate the **nuclear photo-dissociation problem**?





n's + seed $\rightarrow$ heavy (A = 100 − 200)
seeds ($A = 50 - 100$)
^{4}He($\alpha\alpha,\gamma$)^{12}C
^{4}He($\alpha n,\gamma$)^{9}Be
$2n + 2p \rightarrow \alpha$
$\nu_e + n \rightarrow p + e^-$
Alpha Effect
Fuller & Meyer 1995
Meyer, McLaughlin & Fuller 1998
r − process
$T \approx 0.25$ MeV
$T \approx 0.75$ MeV
WFO
$T \approx 0.9$ MeV
r v
ρ T
Heating Region
$\nu_e + n \rightleftharpoons p + e^-$
$\bar{\nu}_e + p \rightleftharpoons n + e^+$
$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$
Proto − Neutron Star



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T~ 0.1 MeV
T~ 0.75 MeV
Alpha Particle Formation
Time
PROTON
NEUTRON

R-Process Nucleosynthesis

Heavy element abundance determinations in Ultra Metal-Poor halo star CS 22892-052

$$\left[Fe/H\right] \approx -3.1$$

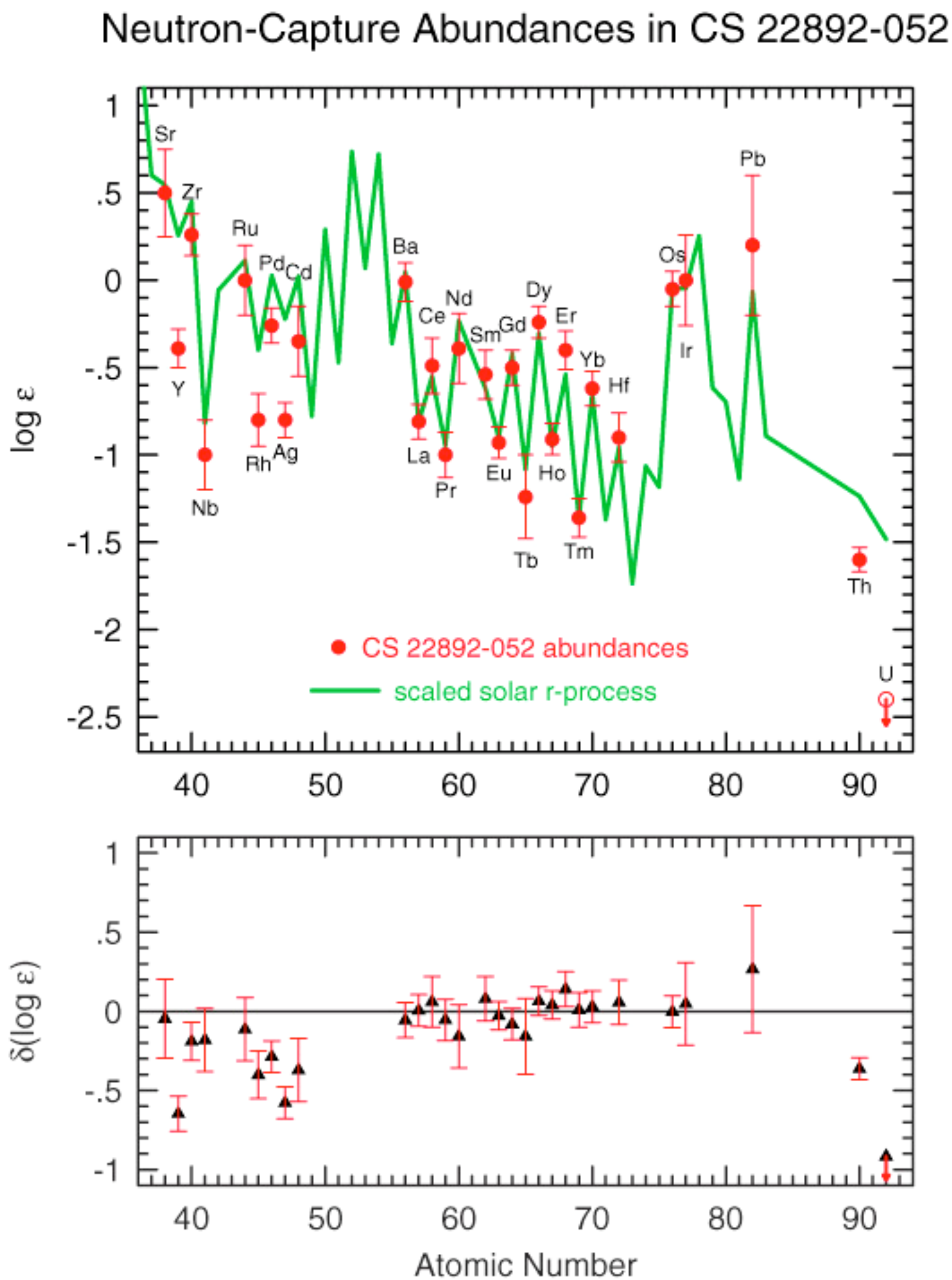
Mass A>100 abundance pattern fits that of solar system, lower nuclear mass material has an abundance pattern which does not, in general, fit the solar pattern. This trend is evident in other Ultra Metal-Poor Halo (UMP's) stars as well.

The same pattern is seen in several other UMP's. Qian & Wasserburg believe that these r-process nuclides were deposited on the surfaces of these otherwise quiescent stars by companions that became core collapse supernovae.

universal abundance pattern?

A = 130 & 195 peak have comparable abundances. Why?

Fission cycling in neutron-rich conditions? (McLauglin & Buen 05)



Necessary and sufficient conditions for getting an r-process that can reproduce the observed abundance pattern.

Necessary: we start with a "seed" nucleus like Fe (mass ~ 100) and we need to capture enough neutrons to get out to uranium (mass ~ 200). So we need something like 100 neutrons per seed nucleus.

Furthermore, we need to do this neutron capture in an environment that produces and maintains the abundance peaks at A = 130 and 195. We also must reproduce the observed rate of r-process production in the Galaxy.

Sufficient: we must reproduce the observed abundance pattern in detail. We need to know the r-process path and nuclear weak rates and masses along this.

If we know the baseline nuclear physics sufficiently well, then we can use the measured abundance pattern to infer, e.g., neutrino exposure and neutrino energies that may give clues as to the r-process site and new neutrino physics.

The Alpha Effect

The paradox of neutrino-heated *r*-Process nucleosynthesis

Require neutrino interactions on free nucleons to give enough energy to each baryon to overcome the gravitational binding energy near the neutron star (~100 MeV per baryon). Since the average energies of neutrinos are ~ 10 MeV, we need some ~10 neutrino and antineutrino captures per nucleon to ensure ejection of the material.

However, formation of alpha particles incorporates all protons thereby isolating some free neutrons. These can capture electron neutrinos to become protons, which are immediately incorporated into alpha particles. Each reaction $\nu_e + n \rightarrow p + e^-$ takes out two neutrons!

In short order there are not enough neutrons to make the *r*-Process

(Fuller, G. McLaughlin, B. Meyer)